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Remanufacturing lot-sizing under alternative perceptions of returned units' quality

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Abstract

One of the critical parameters in reverse supply chain management is the increased variability of the quality condition of used, returned products. The volatile nature of returns' quality often dictates the establishment of quality assessment procedures and the development of technologies that facilitate the fast, accurate and inexpensive classification of returns. The appropriate degree in which a firm has to allocate resources for acquiring information on the quality of returned units, naturally, depends on the anticipated improvement of recovery activities' profitability. Therefore, the quantification of the savings associated with confronting or resolving quality uncertainty is a necessary input during the determination of the proper recovery procedures' configuration. In the current paper, we study a remanufacturing system in a multi-period setting in which returns' quality information is exploited during remanufacturing planning. However, in the decision-making process, certain aspects of the problem examined, such as the quantification of shortage cost, are overlooked or simplified. The objective is to examine the advisability of acquiring advanced quality information in order to be used during sub-optimal decision-making processes, in comparison with alternative policies which do not take explicitly into account returns' quality information. Moreover, through an extensive numerical analysis we examine the implications of alternative considerations regarding returned units' quality on remanufacturing planning, lead-time and service-levels and evaluate their impact on the overall system operational cost.

Keywords: logistics policies, quality of returns, inventory management, stochastic lead-time

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1 INTRODUCTION

During the last decades, industry practices and academic research have focused on the issue of recovering the value that remains in products after the end of their use. Apart from the worldwide introduction of legislation for waste reduction, the savings associated with the improved management of commercial returns and the economic benefits from used products' exploitation have rendered reverse supply chain management a key issue in the development of corporate competitive advantage.

The already abundant list of successfully remanufactured products is continuously enriched to include electronic and electric equipment, office automation machinery, vehicle engines and tires, power tools, etc. This product diversity combined with the multiplicity of recovery alternatives create a wide variety of applications with distinctive characteristics, ranging from direct reuse of slightly used commercial returns, at the one end, to high-touch remanufacturing of expensive modules which are consequently reassembled in complex new products, at the other.

In the whole range of value recovery activities, a major complicating factor is their management and planning under increased quality uncertainty. In the related literature, there are numerous examples of the efforts of original equipment manufacturers (OEM) and remanufacturers for the quality assessment of returned products, which aim at improving reverse supply chain management, supporting remanufacturing planning and promoting forward and reverse channels' coordination (see for example Van Wassenhove and Zikopoulos, 2011). In order to determine the extent of necessary investments for returns' quality assessment, it is important to accurately evaluate the financial benefits resulting from operating under different availability levels of information concerning returns' quality condition.

In the present paper we study a remanufacturing system facing uncertainty regarding quality of returns. The variable quality of returned units affects the necessary recovery time and hinders the efficient remanufacturing planning and inventory management. It is assumed that remanufacturing process planning is performed subject to a service-level constraint rather than to a more involved, yet accurate policy, which would explicitly take into account shortage cost. Inventory management and production planning under predetermined service-level is a common approach in industrial applications because of the difficulties in the accurate evaluation of stock-out cost. Our objective is to evaluate the savings related to explicitly taking into account the inherent quality variability during sub-optimal remanufacturing planning, as opposed to alternative policies that rely on incomplete quality information. The alternative policies studied in the current paper occur from simple heuristics that are often employed in practice to copy with information unavailability. Thus, we study the extent in which the simplifying assumptions commonly used during process planning and inventory management cancel the benefits from quality information, in favor of policies that ignore quality uncertainty.

The remainder of the paper is organized as follows: Section 2 reviews the related literature under both reverse and conventional (forward) supply chain context and further clarifies the contribution of the current work. Section 3 presents the model setting and assumptions. Sequentially, in Section 4 we examine a policy that relies on explicit consideration of quality information, as well as, three alternative policies with rather simplistic conceptions of returns' quality. Section 5 contains a numerical investigation and discussion of the most important findings. Finally, Section 6 concludes the paper and proposes directions for future research.

2 LITERATURE REVIEW

The present paper examines a remanufacturing planning and inventory control problem under an infinite horizon setting. Specifically, we consider a remanufacturing facility facing deterministic demand and stochastic lead-time per remanufacturing lot. Lead-time uncertainty is attributed to the variable quality of remanufacturable units, which influences the actual remanufacturing time per lot.

There are a lot of contributions in existing literature that examine alternative policies for production and remanufacturing planning in a multi-period setting, which aim to balance process set-up, inventory holding and stock-out costs. Van der Laan et al. (1999), Inderfurth and Van der Laan (2001) and Mahadevan et al. (2003) are examples of contributions that fall into this context, while Fleischmann et al. (1997) present a review of earlier relevant contributions. More recently, Konstantaras et al. (2010) and Yoo et al. (2012) examine procurement, sorting and recovery policies for hybrid manufacturing / remanufacturing facilities. A common characteristic of the aforementioned contributions is that the stochastic nature of used units' quality is not taken into account explicitly. A problem setting with uncertain quality of returns is considered in Nenes et al. (2010), in which alternative ordering and remanufacturing policies are examined assuming stochastic demand but with deterministic remanufacturing lead-time.

On the other hand, the stream of reverse supply chain literature focusing on the impact of uncertainty regarding returns' quality on remanufacturing advisability, usually pertains on the optimization of procurement decisions without examining explicitly the remanufacturing environment and the implications of imperfect quality on lead-time and production planning. Examples of contributions under this context include Tagaras and

Zikopoulos (2008), Ferguson et al. (2009), Van Wassenhove and Zikopoulos (2010), Teunter and Flapper (2011) and Robotis et al. (2012). A more detailed review of the literature on inventory management and remanufacturing planning can be found in Akcali and Cetinkaya (2011).

Among the limited number of existing contributions which depart from the constant remanufacturing lead-time assumption, Bayindir et al. (2006) examine alternative inventory control policies for remanufacturable units taking into account the impact of different recovery effort on remanufacturing lead-time. Specifically, Bayindir et al. (2006) consider remanufacturing lead-time as a decision variable and assume that increasing lead-time improves recovery process's yield. The contributions of Aras et al. (2004) and Tang and Grubbstrom (2005) are closely related to the work in the present paper, since they examine the issue of stochastic lead-time in remanufacturing. In the former paper, although the assumption regarding the impact of returns' quality on remanufacturing time is similar to the one adopted in the present paper, the main objective is to study the implications of returns' classification on system's profitability. In Tang and Grubbstrom (2005), stochastic lead-time for remanufacturing and new product acquisition is considered as a consequence of the uncertainty that generally characterizes industrial processes. On the contrary, in the present paper remanufacturing lead-time is a result of the different recovery effort and remanufacturing time that is necessary for units of different quality condition.

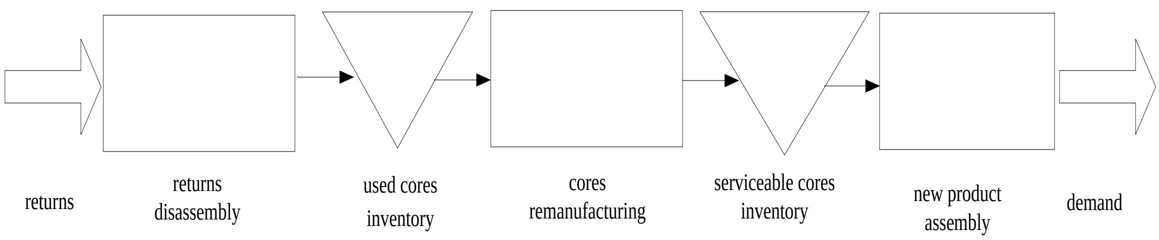
The issue of stochastic lead-time and its impact on production and inventory management decisions has been addressed in a number of papers that fall into conventional supply chain management context. Two of the earliest relevant contributions are those of Liberetone (1979) and Spichas (1982). Paknejad et al. (1992), He et al. (1998), Ben-Daya and Hariga (2004) and Kim et al. (2005) are examples of more recent related contributions.

Summarizing, the most important novel characteristic of the problem setting considered here in comparison with existing contributions, is the origin of the uncertainty in the procurement / manufacturing lead-time. Specifically, variable lead-time is attributed to the different quality states of the returned units that compose the remanufacturing lot. Moreover, our analysis evaluates the impact of different types of available quality information on system's economic outcome taking into account certain simplified process planning and inventory management approaches, which are commonly adopted in industrial applications.

3 MODEL DESCRIPTION AND ASSUMPTIONS

We study a remanufacturing facility, which recovers subassemblies (cores) from used products, and consequently remanufactures and assembles them into new products in order to satisfy demand in a multi-period setting. Figure 1 depicts the remanufacturing and assembly process. It is assumed that there is ample quantity of returned products, which upon their return are disassembled to acquire the cores. Used cores are consequently stored in used cores' inventory. Inventory holding cost at this stage, is considered negligible. All returned cores are suitable for remanufacturing, and therefore they can be used for satisfying assembly requirements, which have a deterministic rate equal to D units per year. However, each core can be in one of two alternative quality states, good (type 1) or poor (type 2) quality state. The proportion of type 1 cores in the remanufacturing quantity, q, is a random variable with known probability density and cumulative probability functions, denoted by $g(q)$ and $G(q)$, respectively. The only assumption regarding the distribution of random variable q is that it is an invertible, continuous function on the (0, 1) interval. When required, a remanufacturing order is released and a batch of Q used cores is taken from inventory in order to undergo remanufacturing. Each order-release creates a fixed ordering cost, c_p, which is associated with the time and effort for remanufacturing set-up. This cost is assumed independent to the value of Q. Remanufacturing time per core is deterministic, but differs according to each core's quality condition. Specifically, remanufacturing of type 1 or 2 cores requires t_{r1} or t_{r2} time units, respectively. It is natural to assume that type 1 cores' remanufacturing lasts less than type 2 cores' remanufacturing, i.e., $t_{r1} < t_{r2}$. Apart from different remanufacturing times, we assume that there is no other difference between the two core types, and therefore remanufactured cores are identical, regardless of their initial condition.

Figure 1. Remanufacturing and assembly process



Upon completion of remanufacturing of the entire lot, the remanufactured cores are transferred and stored in the serviceable inventory. The serviceable inventory is subject to holding cost, equal to c_h monetary units per core and per year. We assume that the transportation of remanufactured cores occurs only after completion of entire lot's remanufacturing, while remanufacturing orders are released when serviceable cores inventory position drops to the re-order point, *s*.

Given the uncertainty in returns' quality, the remanufacturing time of each batch is a random variable. Therefore, the remanufacturing facility may face stock-outs or excessive inventory holding cost, depending on the selection of *s*. We assume that there is a specific cost associated with every stock-out instance, c_s, to account for the delays and the necessary rearrangements in the assembly process for coping with remanufactured cores' unavailability. Note that there are no lost sales since assembly requirements during stock-outs are met as soon as availability of remanufactured cores is restored. In order to avoid extensive shortage cost, the remanufacturer takes into account returns' quality information in order to maintain a predetermined service-level constraint. Specifically, the probability of a stock-out in each remanufacturing cycle must not exceed α_0 ($0 < \alpha_0 < 1$). This assumption is very common in practical situations, where the lack of sufficient data impedes the evaluation of shortage cost.

4 ANALYSIS

In the subsections that follow, we present separately the analysis of four alternative cases that differ in the type of available quality information. Firstly, we examine the remanufacturing decisions for the case that quality uncertainty is treated explicitly but shortage cost is taken into account only through a predetermined service-level constraint. Sequentially, we examine the case in which quality information is incomplete. Under this assumption, we examine three alternative situations, which differ in the type of available quality information.

4.1 Explicit consideration of quality uncertainty

We assume that the ratio of the two quality-type cores in the remanufacturing batch is a random variable *q*. Specifically, the proportion of class 1 units is equal to *q*, and consequently class 2 units account for the *(*1-*q)* proportion of the batch. Therefore, the number of class 1 units in the remanufacturing quantity *Q*, equals $q \cdot Q$, while the remaining $(1-q)\cdot Q$ units are class 2 units. For a given value of *Q*, the required remanufacturing time, *t,* is a random variable that depends on the exact value of *q* in a batch and equals

$$t(q) = Q\cdot[t_{r1}\cdot q + t_{r2}\cdot(1-q)] = Q\cdot T(q),$$

where, $T(q) = t_{r2} + (t_{r1} - t_{r2})\cdot q$.

Therefore, although demand is deterministic, because of quality variation, demand during the (stochastic) remanufacturing time is a random variable given by:

$$Q\cdot D\cdot T(q).$$

The remanufacturing time is the lead-time that is necessary for the replenishment of the serviceable cores inventory. Since the probability of a stock-out in each remanufacturing cycle depends on the value of lead-time, the desired re-order point, s_0, is evaluated with respect to the required service-level constraint. Specifically, for the stock-out probability must hold:

$$P[s_0 \le Q\cdot D\cdot T(q)] = \alpha_0.$$

The service-level constraint can be rewritten in terms of lot's quality, *q*, as

$$P[\,q \le q_0\,] = \int_0^{q_0} g(\,q\,)dq = \alpha_0,$$

and therefore, a critical proportion value ensuring the required service-level can be evaluated by

$$q_0 = G^{-1}(\,\alpha_0\,), \tag{1}$$

where $G^{-1}(.)$ is the inverse of the cumulative distribution function of *q*. The re-order point, s_0, is evaluated then using:

$$s_0 = Q\cdot D\cdot[\,t_{r2} + (\,t_{r1} - t_{r2}\,)\cdot q_0\,] \tag{2}$$

and the expected number of stock-outs per year is given by

$$E(\text{Number of stock-outs per year}) = \frac{D}{Q}\cdot\alpha_0 = \frac{D}{Q}\cdot\int_0^{q_0} g(q)dq.$$

For evaluating the serviceable inventory holding cost, it is necessary to compute the expected value of on-hand inventory just before remanufacturing completion. If it is assumed that the duration of stock-outs is negligible compared to the cycle duration, it is possible to equate the expected on-hand inventory to the expected net stock just before production completion. Thus,

$$E(OH_L) \approx \int_0^1 [s_0 - Q \cdot D \cdot T(q)] \cdot g(q)dq = s_0 - Q \cdot D \cdot t_{r2} - Q \cdot D \cdot (t_{r1} - t_{r2}) \cdot E(q),$$

which because of (2), yields

$$E(OH_L) = Q \cdot D \cdot (t_{r1} - t_{r2}) \cdot [q_0 - E(q)].$$

Given that the on-hand inventory during any cycle varies between $Q + E(OH_L)$ and $E(OH_L)$ the average on-hand inventory is computed by

$$E(OH) = \frac{E(OH_L) + [E(OH_L) + Q]}{2} = Q \cdot D \cdot (t_{r1} - t_{r2}) \cdot [q_0 - E(q)] + \frac{Q}{2}.$$

The annual average holding cost is given by the product of unit holding cost and the average inventory on hand. Subsequently, we can write the expected total cost function, as a summation of the fixed, inventory holding and stock-out costs:

$$E[TC_0(Q)] = c_p \cdot \frac{D}{Q} + c_h \cdot \frac{Q}{2} + c_h \cdot Q \cdot D \cdot (t_{r1} - t_{r2}) \cdot \left[q_0 - E(q)\right] + c_s \cdot \frac{D}{Q} \cdot \alpha_0. \quad (3)$$

The first two terms of (3) stand for the annual set-up and the cycle inventory holding costs, respectively. The third term is the holding cost for the inventory that is necessary for maintaining the required service-level. Finally, the last term of (3) is the expected stock-out cost per year.

The first-order derivative of (3) is given by

$$\frac{\partial E[TC_0(Q)]}{\partial Q} = - c_{p1} \cdot \frac{D}{Q^2} + \frac{c_h}{2} + c_h \cdot D \cdot (t_{r1} - t_{r2}) \cdot \left[q_0 - E(q)\right] - c_s \cdot \frac{D}{Q^2} \cdot \alpha_0,$$

which vanishes for

$$Q_0^* = \sqrt{\frac{2 \cdot \left(c_p + c_s \cdot \alpha_0\right) \cdot D}{c_h \cdot \left\{1 + 2 \cdot D \cdot \left(t_{r1} - t_{r2}\right)\left[q_0 - E(q)\right]\right\}}}. \quad (4)$$

Checking the second-order derivative of $E[TC_0(Q)]$, we conclude that the first-order conditions are both necessary and sufficient for Q_0^* to minimize the expected total cost and thus, Q_0^* is the optimal remanufacturing quantity. For typical values of required service-level, or equivalently for typical values of shortage cost, the value of q_0 does not exceed $E(q)$, and therefore it holds that

$$D \cdot \left(t_{r1} - t_{r2}\right) \cdot \left[q_0 - E(q)\right] \geq 0 > -0.5,$$

which is a necessary condition for the quantity under the square root sign to be positive.

4.2 Ignoring quality uncertainty

The alternative scenarios considered deal with the case that the remanufacturer does not have complete information or they decide not to take into account the uncertainty in returns' quality. That is, remanufacturing decisions are based on the simplifying assumption that remanufacturing time per lot is deterministic. In order to increase the applicability of our analysis, we study three alternative cases:

Case 1: the conservative case, in which the remanufacturing time is assumed equal to class 2 units remanufacturing time.

Case 2: the expectation case, in which remanufacturing time is assumed equal to the expected remanufacturing time, taking into account only the expected value of the proportion of the quality classes in the remanufacturing lot, *E(q)*.

Case 3: the median case, in which we assume that remanufacturing time for the two quality classes is known, but the ratios of the two classes in the remanufacturing quantity are assumed equal.

In each of the three alternative cases, remanufacturing planning considers a single value of the quality ratio, q_i, i = 1, 2, 3, ignoring the true distribution and variability of the random variable. Hence, the re-order point can be determined using:

$$s_i = Q \cdot D \cdot [\, t_{r2} + (t_{r1} - t_{r2}) \cdot q_i \,] \text{, } i = 1, 2, 3. \quad (5)$$

Below we explain in more detail the alternative policies considered in the present paper and the rationale for their selection.

Case 1 (the conservative case)

The remanufacturer is aware that there is some uncertainty regarding the time required for remanufacturing a used core. Because of the unavailability of any formal analysis regarding the composition of the lot and the necessary remanufacturing time, the largest remanufacturing time observed, t_2, is used for remanufacturing planning in order to avoid any chance of a stock-out. For this policy, the shortage cost vanishes, at the cost of holding excess inventory. According to the aforementioned assumptions, in this case holds:

$q_1 = 0$, *and*

$T(q_1) = t_{r2}$.

A related policy would be the one that considers only the lowest remanufacturing time. Such policy would be advisable in cases with large holding cost relatively to stock-out cost.

Case 2 (the expectation case)

The differences in remanufacturing time have attracted the attention of the remanufacturer and following a simple analysis of the completion times of a number of remanufacturing lots, the average remanufacturing time, $E(T)$, can be evaluated directly. Of course, $E(T)$ is actually a function of t_{r1}, t_{r2} and $E(q)$. The resulting policy does not take into account the stock-out probability but eliminates the part of inventory holding cost, which is related to uncertainty of the lot's remanufacturing time. Based on Case 2 assumptions, it holds that:

$q_2 = E(q)$, *and*

$T(q_2) = t_{r2} + (t_{r1} - t_{r2}) \cdot E(q)$.

Case 3 (the median case)

In this case, it is assumed that although the exact values of the remanufacturing times for both of the available quality classes are known, the remanufacturer ignores the ratio of the two classes in the remanufacturing quantity. Therefore, given that the only available information is that the range of possible values is in the (0, 1) interval, the median value of the quality distribution domain is adopted, i.e.,

$q_3 = 0.5$, *and*

$T(q_3) = 0.5 \cdot (t_{r1} + t_{r2})$.

Of course, it is possible to a select different q_3 value, if quality information that dictates so is available.

4.3 Analysis of incomplete quality information policies and comparison

In any of the three alternative cases, the remanufacturing quantity and the re-order point are determined based on the assumption of deterministic remanufacturing time, $T(q_i)$. The expected total cost is given by:

$$TC_i(Q) = c_p \cdot \frac{D}{Q} + c_h \cdot \frac{Q}{2} \text{, } i = 1, 2, 3. \quad (6)$$

Note that (6) is simply the classical *EOQ* total cost function which is minimized for:

$$Q_i^* = Q^* = \sqrt{\frac{2 \cdot c_p \cdot D}{c_h}} \text{, } i = 1, 2, 3. \quad (7)$$

For deterministic lead-time demand, the re-order point is computed by

$$s_i = Q \cdot D \cdot T(q_i) \text{, } i = 1, 2, 3, \quad (8)$$

and the resulting service-level equals

$\alpha_i = G(q_i)$, i = 1, 2, 3.

Given that the actual remanufacturing time depends on the random variable q, the true value of total expected cost is computed by

$$E[TC_i(Q)] = c_p \cdot \frac{D}{Q_i}$$
$$+ c_h \cdot \left\{ \frac{Q_i}{2} + Q \cdot D \cdot (t_{r1} - t_{r2}) \cdot \left[q_i - E(q) \right] + \frac{Q_i \cdot D^2 \cdot (t_{r1} - t_{r2})^2}{2} \cdot \int_0^{q_i} (q - q_i)^2 g(q) dq \right\}$$
$$+ c_s \cdot \frac{D}{Q_i} \cdot \alpha_i \text{, } i = 0, 1, 2, 3. \tag{9}$$

It should be noted that in (9), for the evaluation of on-hand inventory is used the accurate expression rather than the simplifying assumption used in the formulation of (3) and for obtaining the policy parameters. Since remanufacturing planning ignores the true distribution and variability of quality and accounts only for a specific q_i value, the remanufacturing policy parameters (Q, s) are determined using (7) and (8) respectively. However, only the first two terms of (9) account for the *EOQ*-related costs, while the last three are due to the uncertainty in returns' quality. By the comparison of (4) and (7), it is evident that in general $Q_0^* \neq Q_i^*$. Therefore, the first two terms of (9) are larger for the complete information policy. On the other hand, the benefits of exploiting quality information derive from the fact that the tradeoff between safety stock and stock-out costs is taken into account. The inventory that is held to protect the remanufacturer against shortages is maximized for the conservative policy and vanishes for the expectation policy. The median case yields zero safety inventory only for quality distributions with $E(q)$ = 0.5. Of course, the extent of the savings associated with the complete information policy depends on the selection of the service-level criterion.

5 NUMERICAL ILLUSTRATION AND RESULTS

In the current section we examine numerically a number of different scenarios in order to illustrate the findings of the preceding analysis and to study the impact of problem parameters on the operational cost of the alternative policies. In the scenarios studied, we consider three different values of the expected value of quality class 1 proportion in the remanufacturing quantity. Moreover, each $E(q)$ value is examined at three levels of quality variance (i.e., small, medium and high), resulting in a total of 9 different cases. In every case, we assume that the quality proportion, q, follows a Beta distribution with probability density function given by

$$g(q) = \frac{q^{a-1} \cdot (1-q)^{b-1}}{B(a,b)},$$

where *B(a,b)* is the Beta function with parameters *a*, *b*.

The mean and variance of the Beta distribution are determined by the parameters *a* and *b* using

$$E(q) = \frac{a}{a+b} \text{, and}$$

$$Var(q) = \frac{a \cdot b}{(a+b)^2 \cdot (a+b+1)},$$

respectively. The nine different combinations of *a* and *b* parameters are reported in Table 1, along with the resulting mean and variance values.

Table 1: Alternative *a* and *b* combinations and resulting expectation and variance values

#	*a*	*b*	*E(q)*	*Var(q)*
1	1.0	3.0	0.25	0.038
2	2.0	6.0	0.25	0.021
3	3.0	9.0	0.25	0.014
4	2.8	2.8	0.50	0.038
5	5.5	5.5	0.50	0.021
6	8.5	8.5	0.50	0.014
7	3.0	1.0	0.75	0.038
8	6.0	2.0	0.75	0.021
9	9.0	3.0	0.75	0.014

Although it is not possible to conduct a numerical experiment for every possible combination, the selection of the values examined takes into account the usual relationships among the cost parameters in real-life applications, as well as the commonly employed service-level values. Specifically, the numerical analysis is carried out for two alternative values of the set-up cost, c_p, namely 1,000 and 750, two values of annual requirements for remanufactured units, D, namely 3,000 and 5,000 units per year, and two values of the desired service-level, $1-\alpha$, equal to 95% and 99%, respectively. The remainder cost parameters are defined proportionally to c_p, while remanufacturing time per quality class is defined with respect to D. Specifically, we assume $c_h = 1\% \cdot c_p$ or $c_h = 10\% \cdot c_p$ and $c_s = 150\% \cdot c_p$ or $c_s = 125\% \cdot c_p$. Remanufacturing time is set equal to $t_{r1} = 60\% \cdot D^{-1}$ or $t_{r1} = 40\% \cdot D^{-1}$ and $t_{r2} = 150\% \cdot t_{r1}$ or $t_{r2} = 175\% \cdot t_{r1}$. The exact values of the parameters used in the numerical examples are reported in Tables 2 and 3.

Table 2: Cost parameters used in the numerical investigation

#	c_p	c_h	c_s
1	1,000	10.0	1,500
2	1,000	10.0	1,250
3	1,000	100.0	1,500
4	1,000	100.0	1,250
5	750	7.5	1,125
6	750	7.5	938
7	750	75.0	1,125
8	750	75.0	938

Table 3: Demand and remanufacturing time per returned unit's class used in the numerical investigation

#	D	$10^5 t_{r1}$	$10^5 t_{r2}$
1	3,000	20	35
2	3,000	20	30
3	3,000	13	23
4	3,000	13	20
5	5,000	12	21
6	5,000	12	18
7	5,000	8	14
8	5,000	8	12

The alternative parameter values presented in Tables 1-3, result in 1152 different scenarios. Each of the alternative scenarios examined is evaluated for the case with explicit consideration of the returns' quality, as well as, for the three alternative considerations, i.e., the conservative, the expectation and the median cases. The determination of the order quantity, Q_i, is carried out using (4) for the system with explicit consideration of quality uncertainty and (7) for the three alternative ones. The critical proportion value for the informative system, q_0, is calculated by (1), while the q_i, i = 1, 2, 3, values are determined according to the analysis of Section 4.2. Finally, the values of $E[TC_i(Q_i)]$ for i = 0, 1, 2, 3, are determined using (9).

The average annual total cost for the case of explicitly considering returns' quality equals 17,885 monetary units per year, and the average cost of the conservative case exceeds this figure by 797 monetary units per year. The average supplementary annual cost of the remainder two cases equals 3,837 and 3,855 monetary units per year, respectively. The average percentage of cost reduction, when quality is considered explicitly relatively to the three alternative cases (i.e., $100 \cdot [E(TC_i) - E(TC_0)] / E(TC_0)$ for i = 1, 2, 3) equals 4.45, 21.46 and 21.56%, respectively. Figure 2 depicts the distribution of the percentage cost improvement when quality information is exploited, as compared with the three alternative systems examined.

As Figure 2 illustrates, in a number of scenarios both the conservative and the median case achieve lower total cost than the informative model. As explained previously, the model with complete quality information aims to counterbalance inventory holding cost to shortage cost. Since c_s value is not taken into account explicitly, it is natural that for certain combinations of problem parameters the base-case model is inefficient compared to some of the alternative policies. However, the degree of this inefficiency is rather small in the scenarios examined.

The average cost reduction when the conservative case is favourable equals 1.32%. Accordingly, in the median case the average reduction equals 0.63%. On the other hand, in the scenarios that it is preferable to account explicitly for returns' quality, average savings equal 1,103, and 4,534 monetary units per year, as compared with the conservative and the median case, respectively. The respective percentages equal 6.07 and 25.72 %.

Figure 2: Distribution of expected cost improvement by exploiting quality information compared to (a) case 1, (b) case 2 and (c) case 3.

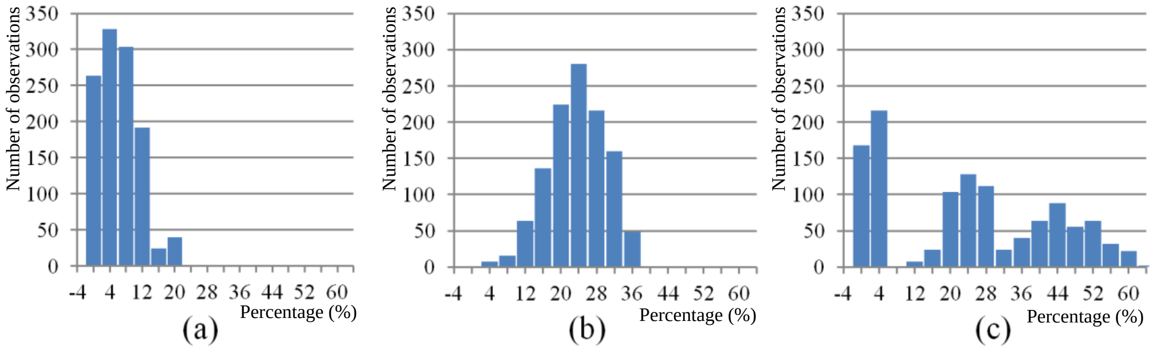


Table 4 summarizes the average total cost values for the base case scenario, with respect to the alternative levels of mean and variance of the random proportion in the remanufacturing lot and of the difference of remanufacturing time per quality class, namely $\Delta t_r = t_{r2} - t_{r1}$, considered in the numerical analysis. Moreover, the percentage of cost increase or reduction, i.e., $100 \cdot [E(TC_i) - E(TC_0)] / E(TC_0)$, when policy *i* (*i* = 1, 2, 3) is used is also reported in Table 4.

Table 4: Expected total cost for base case and relative profitability of alternative policies.

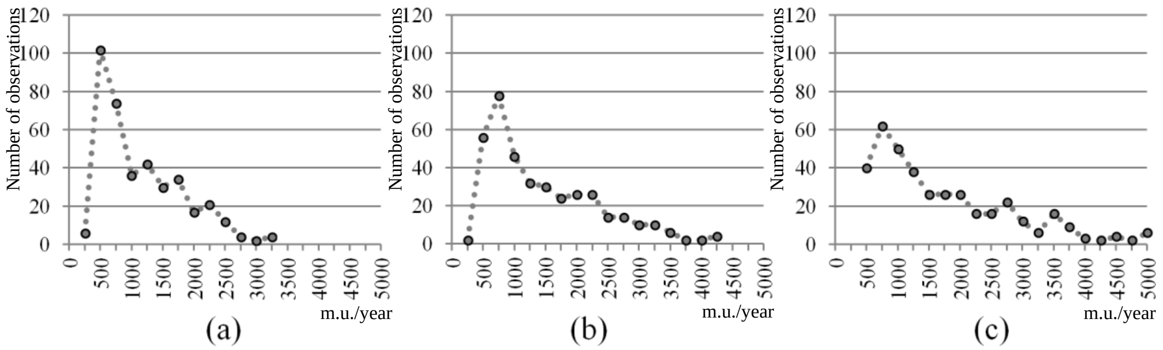
Factor	Level	$E(TC_0)$	$\frac{E(TC_1)-E(TC_0)}{E(TC_0)}\%$	$\frac{E(TC_2)-E(TC_0)}{E(TC_0)}\%$	$\frac{E(TC_3)-E(TC_0)}{E(TC_0)}\%$
E(q)	High	18,202	9.54	15.87	0.20
	Medium	17,905	4.33	21.39	21.39
	Low	17,549	-0.73	27.57	44.04
Var(q)	High	18,200	2.55	19.62	19.91
	Medium	17,832	4.67	21.91	22.19
	Low	17,624	5.92	23.30	23.54
Δt_r	High	18,153	5.62	19.84	20.16
	Low	17,617	3.14	23.38	23.60
Total		17,885	4.45	21.46	21.56

By the numerical results we can conclude that the total expected cost is minimized when *E(q)*, *Var(q)* and Δt_r are low. Although that for *Var(q)* and Δt_r this result is expected, the impact of *E(q)* is counterintuitive and therefore requires further discussion.

Recall that remanufacturing cost (which naturally is non-decreasing in remanufacturing time) is not included in the expected cost function (9). Therefore, the most important implication of *E(q)* on the total cost is through the determination of s_0 and the resulting safety inventory. In order to achieve the desired service-level, the expected value of the quality-classes proportion, *E(q)*, is replaced by a more conservative value, q_0. As our numerical analysis indicates, for lower values of *E(q)* the value of q_0 corresponds to lower safety stock investments, compared to the cases with high *E(q)* value. In other words, *E(q)* - q_0 is decreasing in *E(q)*. Thus, the safety-stock holding cost increases in *E(q)*, resulting in increased overall cost. Figure 3 illustrates this feature for the numerical examples examined. It is clear that as *E(q)* increases, the mass of the distribution of the safety-stock cost shifts towards the right. Of course, if remanufacturing cost is taken into account, depending on the difference between quality type 1 and 2 remanufacturing costs, the relevant profitability may be reversed.

Another interesting finding is that *E(q)* demonstrates an ambiguous impact on the relative advisability of policies 1 and 3 compared with the one which implicitly takes into account quality uncertainty. This property is related to the relationship between q_0 and q_i values. As the difference between q_0 and q_i increases, policy *i* becomes less advisable, as compared to the complete information case. For example, for *E(q)* = 0.25, q_0 approaches q_1 while for *E(q)* = 0.75, q_0 is close to q_3. Note that in Figure 2(c), the scenarios with improvement equal to 0 $\pm$ 4% are the 384 scenarios with *E(q)* = 0.75.

Figure 3: Distribution of the values of $Q \cdot D \cdot (t_{r1} - t_{r2}) \cdot [q_i - E(q)]$ for the informative policy for values of *E(q)* equal to (a) 0.25, (b) 0.50 and (c) 0.75.



As the results of the numerical analysis indicate, lower variability favors more prominently the accurate model with respect to the three alternative policies. The explanation for this phenomenon is that the necessary safety stock increases in quality variability. Therefore, the financial benefits of the informative policy decrease compared to the 3 alternative cases that do not take into account service-level at all.

Summarizing, as it can be inferred by the numerical examples presented, the efficiency of the complete information policy can be significant, even though shortage cost is not considered explicitly. However, the selection of the most efficient policy is not possible without performing previously an analysis for evaluating quality characteristics and cost parameters for the application in question. In other words, it may be advisable in certain cases not to exploit complete quality information during remanufacturing planning, but at the same time, the firm must have enough quality information in order to decide not to use it. Among the alternative policies examined, the conservative one exhibits better performance which is more prominent for lower values of the c_h / c_s ratio. The median case can result in good results, but only when there is adequate information on expected yield. Therefore, when shortage cost is taken into account only implicitly through an arbitrary service-level criterion, the benefits from information on the exact quality distribution may be rather small; adequate performance can be accomplished by evaluating accurately the different remanufacturing times of available quality classes and their percentage in the return's lot. On the other hand, a policy exploiting information only on the *E(q)* value does not provide satisfactory results.

6 CONCLUSION

In the present paper, the issue of quality uncertainty in returned used products is examined and the potential savings obtained from explicitly taking it into account are evaluated. Our analysis provides evidence that the additional benefits of advanced quality information may vanish when simple process planning and inventory management policies are employed. Moreover, although a policy that relays exclusively on the expectation of returns' quality distribution is not advisable, this information is decisive for the selection of the suitable policy. The expected value of the proportion of good quality returns defines which alternative with incomplete information would be advisable. The numerical study indicates that under our assumptions, information on the distribution of the proportion of each quality class in returned quantity is more useful when quality variability is small.

An open issue is the examination of a problem with more that two returns' quality classes in order to examine if our findings carry over. Another interesting extension of the present work is the determination of bounds and indices which would define regions of the problem parameters where a specific alternative policy would be advisable.

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