Int. Journal of Business Science and Applied Management, Volume 7, Issue 3, 2012

Two mathematical formulations for the containers drayage problem with time windows

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Abstract

The containers drayage problem studied here arise in ISO container distribution and collecting processes, in regions which are oriented to container sea ports or inland terminals. Containers of different sizes, but mostly 20ft, and 40ft empty and/or loaded should be delivered to, or collected from the customers. Therefore, the problem studied here is closely related to the vehicle routing problem with the time windows that finds an optimal set of or routes visiting deliveries and pickups customers. The specificity of the container drayage problem analyzed here lies in the fact that a truck may simultaneously carry one 40ft, or two 20ft containers, using an appropriate trailer type. This means that in one route two, three or four nodes can be visited, which is equivalent to the problem of matching nodes in single routes which provide a total travel distance shorter than in the case when nodes are visited separately. The paper presents two optimal MIP mathematical formulations for the case when pickup and delivery nodes could be visited only in specific time interals - time windows. Proposed approaches are tested on numerical examples.

Keywords: containers drayage, pickup delivery, vehicle routing problems with TW

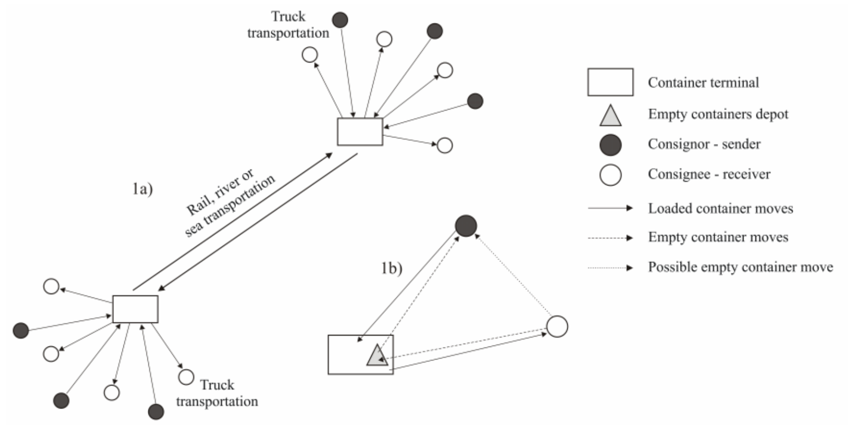
Acknowledgements: This research was partially supported by the Ministry of Education and Science Republic of Serbia, through the projects TR 36002 and TR 36006, for the period 2011-2014.

1 INTRODUCTION

The routing problem studied here is a typical for the intermodal transportation systems where containers are delivered by trucks to customers oriented to a container sea port or an inland container terminal. In the intermodal transportation (Fig. 1a) the major part of the cargo's journey is performed by rail, inland waterway or sea, while the initial and/or final legs, distribution and collection of containers, are typically carried out by road (for more details about intermodal transportation systems see Crainic and Kim, 2007). Truck should deliver a loaded container that has arrived at an terminal (import i.e. inbound container) to a consignee, and to pickup and haul back the loaded container from consignor to the terminal (export i.e. outbound container). In the case when a part of a container terminal serves as an empty containers' depot, in addition to pickup – delivery operations with loaded containers, the empty containers also need to be delivered to a shipper for loading and hauled back empty to the terminal after unloading goods at the consignee site. In addition, when the time of subsequent shipment and the suitability of an empty container at the consignee site, in terms of type size and ownership, are satisfied, it is also possible to move the empty containers directly to a shipper instead of hauling them back to the terminal's depot and having them transferred latter. From there, when considering container transportation within a local region oriented to an intermodal terminal that includes empty containers' depot, few possible types of container moves, also known as drayage operations (Macharis & Bontekoning, 2004), can be recognized (Fig. 1b).

Drayage operations are driven by the need to fulfill customer demands while satisfying various constraints imposed by the technology and customers' requirements. Drayage includes regional movements of loaded and empty equipment (trailers and containers) by tractors between terminals, shippers, consignees, and equipment yards. In general, drayage operations involve not only the provision of containers but also empty trailers (Macharis & Bontekoning, 2004), while in this research only the problem of containers pickup and delivery is considered.

Figure 1: Intermodal transportation system (1a) and possible types of container moves in container truck transportation (1b)



Most intermodal containers are sized according to International Standards Organization (ISO). Based on ISO, containers are classified in several groups (10ft, 20ft, 30ft, 40ft, 45ft and since recently 48ft and 53ft), where 20ft and 40ft containers are the most frequently used all over the world. An important issue in containers pickup and delivery is coordination of the dimensions of road transport vehicles with the dimensions of intermodal containers. In Europe, except Finland and Sweden, and Asia, road vehicles are restricted to transport only 20ft and 40ft containers, only few countries allow 45ft containers, while larger containers are in use only in the USA and Canada (Nagl, 2007). In conjunction with the length, the weight of container is also very important. Most countries allow transport of one fully loaded 20ft or 40ft container, but although transport of two 20ft containers would be possible regarding length, the weight is an obstacle. In most countries transport of two loaded 20ft containers by a standard vehicle is not permitted, except in the case when the weight limitation of 26 tons is not exceeded. In the USA, Australia, Canada, Finland and Sweden the vehicles in use are the ones that offer the possibility of transporting two fully loaded 20ft containers, while the EU has set up regulations which permit certain types of vehicles called "modular concept vehicles", offering the possibility of transporting two fully loaded 20ft containers using special combined chassis. In turn, the use of those technical solutions provides different opportunities for improving the efficiency of container transportation by merging different pickup and delivery operations in a single route.

Drayage operations and especially container truck transportation account for a significant portion of the total transportation cost. Therefore, it is very important to improve the efficiency of container transportation through the optimization of such transportation processes, which leads to necessity of solving the truck scheduling problem in container drayage operation (Zhang et al. 2010).

Optimization of container drayage operation has received increased attention over the past decade due to its importance in intermodal freight transportation. Jula et al. (2005) formulated the problem of container movement with time windows at origins and destinations as asymmetric multiple traveling salesman problem and proposed three solving approaches. Coslovich et al. (2006) investigated a container drayage operation with the present and future operating costs minimized. Imai et al. (2007) formulated a container drayage problem as a pickup and delivery and proposed Lagrangian relaxation to solve the problem. Chung et al. (2007) built several mathematical models of container truck transportation. They formulate the basic problem where every vehicle can transport exactly one container at a time, and the multi-commodity problem with a combined chassis used in transporting two 20ft containers or one 40ft container. To solve the problem a solution algorithm based on the Insertion Heuristic was proposed. Namboothiri and Erera (2008) studied the management of a fleet of trucks providing container pickup and delivery service (drayage) to a port with an appointment-based access control system. Zhang et al. (2010), considered a truck scheduling problem for container transportation in a local area with multiple depots and multiple terminals. They proposed an approach based on an integer programming heuristic determines pickup and delivery sequences for daily drayage operations with minimum transportation cost. Savelsbergh and Sol (1995) show that container transportation problems belong to pickup and delivery problems, and because of the nature of the problem, drayage operations also corresponds to multi-stop Vehicle Routing Problems with Backhauls (VRPB). A more detailed insight in VRPB, as well as in Vehicle Routing Problems with Pickup and Delivery (VRPPD), can be found in recent comprehensive overview given by Parragh et al. (2008a, 2008b).

The purpose of this paper is to propose mathematical formulations for the optimal trucks' routing in containers drayage operations in the case when pickup and delivery nodes may be visited only during a certain predefined time intervals. In this way, our previous research (Vidović et al. 2011a, Vidović et al. 2011b) has been extended by introducing additional mathematical formulation based on general mixed integer programming model for the vehicle routing problem with simultaneous pickups and deliveries (VRP-SPDTW), proposed by Mingyong and Erbao (2010). Therefore, most of introductionary part remained the same, while as in previous research, we consider both, empty and loaded containers' moves in case when combined chassis for transporting two 20ft containers or one 40ft container are used. Direct moves of empty containers from a consignee are to a shipper's, as a relatively rare tasks, are not considered.

In the container drayage operations realized by combined chassis vehicles, VRPBTW refers to the problem where up to four nodes can be visited in a single route starting and ending in container terminal or depot which is assumed here to be part of the terminal.

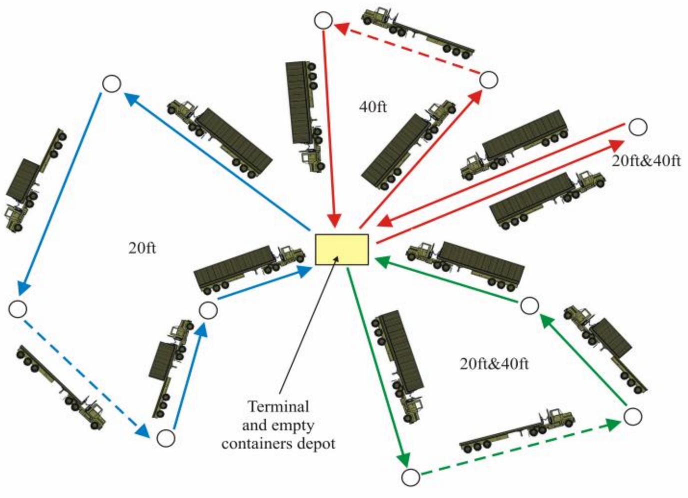
Our research extends the problem analyzed by Zhang et al. (2010) to the multi-commodity case, but for the case when only one intermodal terminal operates in the region. Also, our research extends the problem analyzed by Imai et al. (2007), to the multi-commodity case. Besides respecting the multi-commodity, this paper also differs in the overall objective which is to find optimal matching possibilities of nodes that should be merged in the same route forming backhaul loop. Another characteristic of the proposed formulations is in respecting the simultaneous pickup and delivery operations.

The remainder of this paper is organized as follows. Section 2 presents two optimal problem formulations. Section 3 presents computational results, and Section 4 gives some concluding remarks.

2 PROBLEM FORMULATIONS

The problem of distributing – collecting ISO containers (20ft, and 40ft) may be described as a variant of VRPB in which a truck visits up to four nodes until return to terminal. Loaded containers arrived in terminal (import i.e. inbound containers), or empty containers from the terminal depot should be delivered to customers, and loaded (export i.e. outbound containers), as well as empty containers should be picked up at a customers' sites and hauled back to the terminal. Therefore, when truck tow combined chassis, matching possibilities include all feasible combinations of 20ft, and 40ft containers that should be transported from/to terminal and customers (Figure2). Obviously, as it can be seen from the Figure 2, there are several possible routes realization concepts and it is worthwhile to choose those resulting with minimal length.

Figure 2: Some of routing possibilities when drayage is realized by combined chassis



2.1 Multiple assignment formulation of the container drayage problem

Let $G(N,E)$ be a graph, where N is the set of nodes $i \in N$ with containers move requests, and $E = \{(i, j) \mid i \neq j, i, j \in N\}$ is the edge set. It is assumed that any node may simultaneously have both, containers demand and supply move requests. Number of requests in all nodes $n_i^{20-}, n_i^{20+}, n_i^{40-}, n_i^{40+}$ which correspond to 20ft containers demand (20-) and supply (20+), and 40ft containers demand (40-) and supply (40+) are known in advance. All containers are available at the beginning of the planning horizon (usually one day), and all vehicles start from the terminal. The assumption that node may simultaneously have demand and supply move requests, gives opportunity of transforming graph into the another, in which each node $i \in N$ is replaced with $n_i = n_i^{20-} + n_i^{20+} + n_i^{40-} + n_i^{40+}$ task nodes. In this way all task nodes of the transformed graph, whose indexes are renumerated, have single move requests, either pickup or delivery (20ft, or 40ft containers).

The set of all task nodes with renumerated indexes, can be now partitioned into four subsets, $N^{20+}, N^{20-}, N^{40+}$ *and* N^{40-}. Sets N^{20-}, and N^{40-} contain only delivery, while sets N^{20+}, and N^{40+} include only pickup nodes with 20ft, and 40ft containers respectively. In this network, when using a combined chassis, there are fifteen possible matchings of task nodes into merged routes, and four direct pickup or delivery routes:

Four nodes matchings:
Terminal → -20 → -20 → +20 → +20 →Terminal
Terminal → -20 → +20 → -20 → +20 →Terminal
Three nodes matchings:
Terminal → -20 → -20 → +20 →Terminal
Terminal → -20 → +20 → +20 →Terminal
Terminal → -20 → +20 → -20 →Terminal
Terminal → +20 → -20 → +20 →Terminal
Terminal → -20 → -20 → +40 →Terminal
Terminal → -40 → +20 → +20 →Terminal
Two nodes matchings:
Terminal → -40 → +40 →Terminal
Terminal → -20 → +40 →Terminal
Terminal → -40 → +20 →Terminal
Terminal → -20 → +20 →Terminal
Terminal → +20 → -20 →Terminal
Terminal → +20 → +20 →Terminal
Terminal → -20 → -20 →Terminal
Direct pickup or delivery routes:
Terminal → +20 →Terminal
Terminal → -20 →Terminal
Terminal → +40 →Terminal
Terminal → -40 →Terminal

Then, the container drayage problem with time windows, when combined chassis is used, can be formulated as the following problem of assigning (matching) nodes in the same route which forms backhaul loop.

$$\min \sum_{p\in N^{20-}}\sum_{q\in N^{20-}\wedge p\neq q}\sum_{w\in N^{20+}}\sum_{e\in N^{20+}\wedge w\neq e} y_{pqwe}c_{pqwe} + \sum_{p\in N^{20-}}\sum_{w\in N^{20+}}\sum_{q\in N^{20-}\wedge p\neq q}\sum_{e\in N^{20+}\wedge w\neq e} y_{pwqe}c_{pwqe} +$$
$$\sum_{p\in N^{20-}}\sum_{q\in N^{20-}\wedge p\neq q}\sum_{w\in N^{20+}} y_{pqw}c_{pqw} + \sum_{p\in N^{20-}}\sum_{w\in N^{20+}}\sum_{e\in N^{20+}\wedge w\neq e} y_{pwe}\cdot c_{pwe} + \sum_{w\in N^{20+}}\sum_{p\in N^{20-}}\sum_{e\in N^{20+}\wedge w\neq e} y_{wpe}\cdot c_{wpe} +$$
$$\sum_{p\in N^{20-}}\sum_{w\in N^{20+}}\sum_{q\in N^{20-}\wedge p\neq q} y_{pwq}\cdot c_{pwq} + \sum_{p\in N^{20-}}\sum_{q\in N^{20-}\wedge p\neq q}\sum_{z\in N^{40+}} y_{pqz}\cdot c_{pqz} + \sum_{t\in N^{40-}}\sum_{w\in N^{20+}}\sum_{e\in N^{20+}\wedge w\neq e} y_{twe}\cdot c_{twe} +$$
$$\sum_{t\in N^{40-}}\sum_{z\in N^{40+}} y_{tz}c_{tz} + \sum_{p\in N^{20-}}\sum_{z\in N^{40+}} y_{pz}c_{pz} + \sum_{t\in N^{40-}}\sum_{w\in N^{20+}} y_{tw}c_{tw} + \sum_{p\in N^{20-}}\sum_{w\in N^{20+}} y_{pw}c_{pw} + \sum_{w\in N^{20+}}\sum_{p\in N^{20-}} y_{wp}c_{wp} +$$
$$\sum_{p\in N^{20-}}\sum_{q\in N^{20-}} y_{pq}c_{pq} + \sum_{w\in N^{20+}}\sum_{e\in N^{20+}} y_{we}c_{we} + \sum_{w\in N^{20+}} y_w c_w + \sum_{p\in N^{20-}} y_p c_p + \sum_{z\in N^{40+}} y_z c_z + \sum_{t\in N^{40-}} y_t c_t \tag{1}$$

subject to

$$\sum_{q\in N^{20-}\wedge p\neq q}\sum_{w\in N^{20+}}\sum_{e\in N^{20+}\wedge w\neq e} y_{pqwe} + \sum_{w\in N^{20+}}\sum_{q\in N^{20-}\wedge p\neq q}\sum_{e\in N^{20+}\wedge w\neq e} y_{pwqe} + \sum_{q\in N^{20-}\wedge p\neq q}\sum_{w\in N^{20+}} y_{pqw} +$$
$$\sum_{w\in N^{20+}}\sum_{e\in N^{20+}\wedge w\neq e} y_{pwe} + \sum_{w\in N^{20+}}\sum_{e\in N^{20+}\wedge w\neq e} y_{wpe} + \sum_{w\in N^{20+}}\sum_{q\in N^{20-}\wedge p\neq q} y_{pwq} + \sum_{q\in N^{20-}\wedge p\neq q}\sum_{z\in N^{40+}} y_{pqz} + \sum_{z\in N^{40+}} y_{pz} +$$
$$\sum_{w\in N^{20+}} y_{pw} + \sum_{w\in N^{20+}} y_{wp} + \sum_{q\in N^{20-}\wedge p\neq q} y_{pq} + y_p = 1 \qquad \forall p\in N^{20-} \tag{2}$$

$$\sum_{p\in N^{20-}}\sum_{q\in N^{20-}\wedge p\neq q}\sum_{e\in N^{20+}\wedge w\neq e} y_{pqwe} + \sum_{p\in N^{20-}}\sum_{q\in N^{20-}\wedge p\neq q}\sum_{e\in N^{20+}\wedge w\neq e} y_{pwqe} + \sum_{p\in N^{20-}}\sum_{q\in N^{20-}\wedge p\neq q} y_{pqw} + \sum_{p\in N^{20-}}\sum_{e\in N^{20+}\wedge w\neq e} y_{pwe} +$$
$$\sum_{p\in N^{20-}}\sum_{e\in N^{20+}\wedge w\neq e} y_{wpe} + \sum_{p\in N^{20-}}\sum_{q\in N^{20-}\wedge p\neq q} y_{pwq} + \sum_{t\in N^{40-}}\sum_{e\in N^{20+}\wedge w\neq e} y_{twe} + \sum_{t\in N^{40-}} y_{tw} + \sum_{p\in N^{20-}} y_{pw} + \tag{3}$$
$$\sum_{p\in N^{20-}} y_{wp} + \sum_{e\in N^{20+}\wedge w\neq e} y_{we} + y_w = 1 \qquad \forall w\in N^{20+}$$

$$\sum_{p\in N^{20-}}\sum_{q\in N^{20-}\wedge p\neq q} y_{pqz} + \sum_{t\in N^{40-}} y_{tz} + \sum_{p\in N^{20-}} y_{pz} + y_z = 1 \qquad \forall z\in N^{40+} \tag{4}$$

Error! Objects cannot be created from editing field codes. (5)

$$w_q \geq w_p + s_p + t_{pq} - M_{pq}(1-y_{pqwe})$$
$$w_w \geq w_q + s_q + t_{qw} - M_{qw}(1-y_{pqwe}) \quad \forall p,q,w,e\in N^{20-}\cup N^{20+} \tag{6}$$
$$w_e \geq w_w + s_w + t_{we} - M_{we}(1-y_{pqwe})$$

$$w_q \geq w_p + s_p + t_{pq} - M_{pq}(1-y_{pqw})$$
$$w_w \geq w_q + s_q + t_{qw} - M_{qw}(1-y_{pqw}) \quad \forall p,q,w\in N^{20-}\cup N^{20+}\cup N^{40-}\cup N^{40+} \tag{7}$$

$$w_q \geq w_p + s_p + t_{pq} - M_{pq}(1-y_{pq}) \quad \forall p,q\in N^{20-}\cup N^{20+}\cup N^{40-}\cup N^{40+} \tag{8}$$

$$a_p \leq w_p \leq b_p \quad \forall p\in N^{20-}\cup N^{20+}\cup N^{40-}\cup N^{40+} \tag{9}$$

$$y_{ijkl} = \begin{cases} 1, \textit{if nodes } i,\ j, k \textit{ and } l \textit{ are merged in the same route} \\ 0, \textit{otherwise} \end{cases} \tag{10}$$

$$y_{ijk} = \begin{cases} 1, \textit{if nodes } i,\ j \textit{ and } k \textit{ are merged in the same route} \\ 0, \textit{otherwise} \end{cases} \tag{11}$$

$$y_{ij} = \begin{cases} 1, \textit{if nodes } i \textit{ and } j \textit{ are merged in the same route} \\ 0, \textit{otherwise} \end{cases} \tag{12}$$

$$y_i = \begin{cases} 1, \text{if } \textit{ node i is served in the direct route} \\ 0, \textit{otherwise} \end{cases} \tag{13}$$

Where

p,q,w,e indexes of customer nodes with 20ft containers supply or demand ($p,q \in N^{20-}$, $w,e \in N^{20+}$)

z,t indexes of customer nodes with 40ft containers supply or demand (t∈N40-, z∈N40+)

i,j,k,l indexes of any arbitrary customer nodes

N^{20-} set of 20ft containers delivery nodes

N^{20+} set of 20ft containers pickup nodes

N^{40-} set of 40ft containers delivery nodes

N^{40+} set of 40ft containers pickup nodes

w_p,w_q,w_w,w_e continuous variable indicating the time at which vehicle starts servicing node p,q,w,e respectively

a_p,b_p the left and the right bound of the time window $[a_p.b_p]$ at the node p indicating the time interval when the node is available for the servicing

s_p,s_q,s_w service times at nodes p,q,w respectively

t_{pq},t_{qw},t_{we} travel times between nodes p-q,q-w,w-e, respectively

M_{pq},M_{qw},M_{we} constants used to linearize time windows constraints, $M_{pq}=max[0, b_p+s_p+t_{pq}-a_q]$

c_{pqwe}, c_{pqw}, c_{pq}, c_p costs of visiting nodes in a single route, including costs from/to terminal (0) (number of indexes denote number of nodes merged in the same route), $c_{pqwe}= c_{0p}+c_{pq}+c_{qw}+c_{we}+c_{e0}$

Objective function (1) tries to minimize total transportation costs of all routes that are used for serving all of supply/demand nodes by solving the set of nodes matching problems. Terms 1 - 2 of the objective function (1) define all allowable four, terms 3 – 7 three and terms 8 – 12 all allowable two nodes matchings. Terms 13 – 16 define direct pickup and delivery routes, visiting only one node. Sets of constraint (2) – (5) prohibit multiple visits of the same node, and provide that each node must be visited exactly once, either in a route visiting four, three, two or one customer node. Constraints (6) to (9) are time windows constraints which allow node to be visited only during the interval when the nodes are available for servicing. Constraints (10) to (13) define binary nature of variables.

Obviously, the idea of defining time windows constraint, in this multiple assignment model formulation, is that the arbitrary sequence of nodes visited in the same route should satisfy sequence of constraints indicating that each node in the sequence must be visited during its available time period.

2.2 General MIP formulation of the VRP with simultaneous pickups and deliveries for the container drayage problem

The second mathematical formulation we employed here to solve the container drayage problem is based on the general mixed integer programming model for the vehicle routing problem with simultaneous pickups and deliveries (VRP-SPDTW), proposed by Mingyong and Erbao (2010). This model contained some classical vehicle routing problems as special cases, and here is slightly modified and adjusted with the objective to represent the container drayage problem.

As same as in the previous model, let $G(N,E)$ be a graph, where N is the set of nodes $i \in N$ with containers move requests, and $E = \{(i,j) \mid i \neq j, i,j \in N\}$ is the edge set. Node with index 0 represents the terminal node, while any customer node may simultaneously have both, containers demand and supply move requests. The assumption gives again opportunity of transforming graph into the another, in which each customer node is replaced with task nodes. In this way all task nodes of the transformed graph have single move requests, either pickup or delivery. It is assumed that the transport costs between task nodes which belong to the same customer node may be neglected.

In the model we used the following notation:

n – number of task nodes to be served

$\overline{k}$ – total number of vehicles

Q – vehicle capacity

c_{ij} – transport costs between customer nodes i and j

x_{ijk} -binary decision variable equals to 1 if and only if vehicle k travel from the task node i to the task node j

y_{ij} – integer variable representing number of containers (transport equivalent units – TEU, denoting 20ft containers) picked up from the task nodes up to task node i, and transported in arc $i,j \in E$

z_{ij} – integer variable representing number of containers (TEU) to be delivered after task node i, and transported in arc $i,j \in E$

s_{ik} – time of beginning of service at task node i by the vehicle k

t_i – service time in the task node i

t_{ij} – travel time between task nodes i and j

The model:

$$\min \quad \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{\bar{k}} c_{ij} x_{ijk} \tag{14}$$

st.

$$\sum_{i=0}^{n}\sum_{k=1}^{\bar{k}} x_{ijk} = 1 \quad \forall\ j = \overline{1, n} \tag{15}$$

$$\sum_{i=0}^{n} x_{ijk} - \sum_{i=0}^{n} x_{jik} = 0 \quad \forall\ j = \overline{1, n};\ k = \overline{1, \bar{k}} \tag{16}$$

$$\sum_{j=1}^{n} x_{0jk} \le 1 \quad \forall\ k = \overline{1, \bar{k}} \tag{17}$$

$$\sum_{i=0}^{n} y_{ji} - \sum_{i=0}^{n} y_{ij} = p_j \quad \forall\ j = \overline{1, n} \tag{18}$$

$$\sum_{i=0}^{n} z_{ij} - \sum_{i=0}^{n} z_{ji} = d_j \quad \forall\ j = \overline{1, n} \tag{19}$$

$$y_{ij} + z_{ij} \le Q \sum_{k=1}^{\bar{k}} x_{ijk} \quad \forall\ i = \overline{0, n};\ j = \overline{0, n};\ i \ne j \tag{20}$$

$$s_{ik} + t_i + t_{ij} - M\left(1 - x_{ijk}\right) \le s_{jk} \quad \forall \quad i = \overline{0, n};\ j = \overline{1, n};\ i \ne j;\ k = \overline{1, \bar{k}} \tag{21}$$

$$a_i \le s_{ik} \le b_i \quad \forall\ k = \overline{1, \bar{k}};\ i = \overline{1, n} \tag{22}$$

$$x_{ijkl} \in \{0,1\},\ y_{ij} \ge 0,\ z_{ij} \ge 0,\ i = \overline{0, n};\ j = \overline{0, n};\ i \ne j;\ k = \overline{1, \bar{k}} \tag{23}$$

The objective function (14) seeks to minimize total transport costs. Constraints (15) ensure that each task node is visited exactly once, while constraints (16) guarantee that the same vehicle arrives and departs from each task node it serves. Constraints (17) prevent multiple depart of the vehicle from the terminal, in the same route. Constraints (18) and (19) are flow equations for pick-up and delivery demands, respectively. Constraints (20) prevent vehicle overloading. Constraints (21) and (22) are time windows constraints. Constraints (23) define variables domains.

3 COMPUTATIONAL RESULTS

Testing the quality of proposed approaches to solving the container drayage problem when combined chassis is used has been carried out on the several problem instances.

Our idea was to test the two MIP models on the Solomon VRPTW benchmark problems (http://web.cba.neu.edu/~msolomon/problems.htm). There are six different sets of problem instances. Task nodes are randomly generated in problem sets R1 and R2, clustered in problem sets C1 and C2, and both, randomized and clustered in problem sets RC1 and RC2. Also, problem sets R1, C1 and RC1 have a short scheduling horizon (few costumers per route) while problem sets R2, C2 and RC2 have a long scheduling horizon (many costumers per route). Demand at each task node in original Solomon instances can have up to 50 units. In the problem of container drayage we observe only several containers that need to be picked from or delivered to each task node. Therefore, we have transformed the original Solomon instances to have both pickup and delivery task, to have fewer tasks per each node and to have four types of tasks. Total number of tasks in each node is obtained by dividing the original Solomon demand by 20 and rounding that number to greater integer value. Then, we randomly allocate derived tasks to $N^{20+}, N^{20-}, N^{40+} and\ N^{40-}$.

Solomon instances have 100 task nodes and optimal solution for container drayage problem cannot be obtained from instances with so many task nodes, and therefore we observe following three cases: small scale problems with 10 and 15 task nodes and large scale problems with 50 task nodes (only first 10, 15 or 50 task nodes are observed in each instance). In each case we observe only the first two instances from the problem set (to provide clarity of the results presentation). The results for the small scale problem instances are presented in Table 1 and for the large scale problem instances in Table 2. Mathematical models were implemented through Python 2.6 API of the CPLEX 12.2. on the Intel(R) Core(TM) i3 CPU M380 2.53 Ghz with 6 GB RAM.

Table 1: Results for smaller problem instances

Number of network nodes	**Instance**	**Number of move requests**	**MULTIPLE ASSIGNMENT FORMULATION**		**GENERAL MIP FORMULATION**	
			Solution	**CPU time (sec)**	**Solution**	**CPU time (sec)**
10	C1 01	11	16964	0.01	16964	0.32
	C1 02	11	22427	0.08	22427 *	1800.00
	C2 01	11	34312	0.02	34312	0.63
	C2 02	11	32869	0.01	32869	4.02
	R1 01	11	33572	0.02	33572	0.11
	R1 02	11	31214	0.01	31214	72.75
	R2 01	11	29931	0.03	29931	4.91
	R2 02	11	30128	0.02	30128	2.06
	RC1 01	13	40883	0.03	40883	773.81
	RC1 02	13	46712	0.02	46712	1519.54
	RC2 01	13	50344	0.02	50344	256.76
	RC2 02	13	48646	0.03	48646 *	1800.00
15	C1 01	17	51797	0.05	51797	1.53
	C1 02	17	46853	0.13	46853	743.73
	C2 01	17	42562	0.03	42562	17.66
	C2 02	17	42856	0.05	42856	671.06
	R1 01	18	40310	0.05	40310	1347.19
	R1 02	18	43257	0.17	43257 *	1800.00
	R2 01	18	69170	0.02	69170	60.76
	R2 02	18	55820	0.02	55820	722.27
	RC1 01	19	63943	0.03	63943	26.78
	RC1 02	19	71521	0.05	71652 *	1800.00
	RC2 01	19	74242	0.05	74242 *	1800.00
	RC2 02	19	70742	0.20	73245 *	1800.00

** Solution obtained after 1800 sec of CPU time (gap between primal and dual was greater than zero)*

Table 2: Results for larger problem instances (general MIP formulation cannot solve large scale problem instances in reasonable CPU time)

Number of network nodes	Instance	Number of move requests	MULTIPLE ASSIGNMENT FORMULATION	
			Solution	CPU time (sec)
50	C1 01	59	141884	2.51
	C1 02	59	140580	87.23
	C2 01	59	184631	1.66
	C2 02	59	167359	23.01
	R1 01	60	204776	0.36
	R1 02	60	190796	25.46
	R2 01	60	174357	11.63
	R2 02	60	163288	123.51
	RC1 01	63	256907	1.42
	RC1 02	63	264194	4.84
	RC2 01	63	244955	10.68
	RC2 02	63	244982	78.58

4 CONCLUDING REMARKS

This paper proposes methods for the optimal trucks' routing in containers drayage operations. We consider both, empty and loaded containers' moves in case when combined chassis for transporting two 20ft containers or one 40ft container are used. To solve the problem containers' drayage is formulated as a multiple assignment problem with time windows, as well as the VRP with simultaneous pickups and deliveries. That is, pickup and delivery nodes are visited only in specific time intervals.

Preliminary testing of the proposed approaches shows that the first model seems to be very promising, but more detailed analysis is left for the further phases. This conclusion is drawn from the observation of Wang and Regan (2002), who stated that typical sub-fleet of trucks, consists of less than 20 trucks and is able to handle at most 75 containers a day.

However, having in mind that Imai et al. (2007) have shown that even the simpler version of the container drayage problem - routing problem with full container load (VRPFC) is NP-hard, appropriate heuristics approach development should be important direction of future research. Namely, the problem studied by Imai et al. (2007), in our notation corresponds to the combination of two nodes matching and direct routes realization, and therefore it can be concluded that the container drayage problem considered here is also NP-hard, since one of its parts is NP-hard. Developing adequate software support, more detailed analysis of proposed approach performances and possible adjustments are without any doubt directions for the very near future, but improving algorithms with the other real life constraints (nonhomogenous fleet of vehicles, multiple use of vehicles…) should be an important issue for the future. Also, metaheuristics application is also considered as an important direction of future research.

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