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Performance evaluation of a merge supply system with a distribution centre, two reliable suppliers, one buffer and Erlang lead times

Michael Vιdalis
University of the Aegean, Department of Business Administration
Michalio Bldg, Michalon 8, 82100, Chios, Greece
Telephone: +30 22710 35143
Email: mvid@ath.forthnet.gr

Stelios Koukoumialos
TEI of Larisa, Department of Business Administration
Peripheral Rd., 411 10, Larissa, Greece
Telephone: +30 24210 53692
Email: skoukoum@teilar.gr

Despoina Ntio
University of the Aegean, Department of Business Administration
Michalio Bldg, Michalon 8, 82100, Chios, Greece
Telephone: +30 24650 21948
Email: dntio@teikoz.gr

Georgios Varlas
University of the Aegean, Department of Business Administration
Michalio Bldg, Michalon 8, 82100, Chios, Greece
Telephone: +30 22710 77024
Email: gvarlas@hotmail.com

Abstract

In this work a two echelon merge supply chain is examined. Specifically, two non identical reliable suppliers feed a distribution centre with a shared buffer. The first echelon consists of the distribution centre and the shared buffer, the second echelon includes two non identical reliable suppliers. There is an unlimited supply of materials to suppliers and an unlimited capacity shipping area after the distribution centre. In other words, suppliers are never starved, and the distribution centre is never blocked. The materials are processed by suppliers with rates following the Erlang distribution. The distribution centre has a reliable machine that pushes material with service times following the Erlang distribution. Blocking appears when one or more suppliers finish their process and try to feed the buffer that is full. The supply network is modelled as a continuous time Markov process with discrete states. The structures of the transition matrices of those systems are explored and a computational algorithm is developed. Our aim is to generate stationary distributions for different values of system's parameters so as the various measures of the system can be estimated. Finally, for the mathematical programming model and the rest of the calculations the Matlab software is used.

Keywords: supply chain management, merge systems, performance measures, Markov processes

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1 INTRODUCTION

A supply chain consists of several possible stages, where goods are produced, transformed, assembled and distributed in order to cover the customer demand. Supply chain networks are distinguished by the pull and push processes. Supply chain management (SCM) is the management of flows (products, capital and information) among the stages of the supply chain, aiming to maximize the total expected profitability.

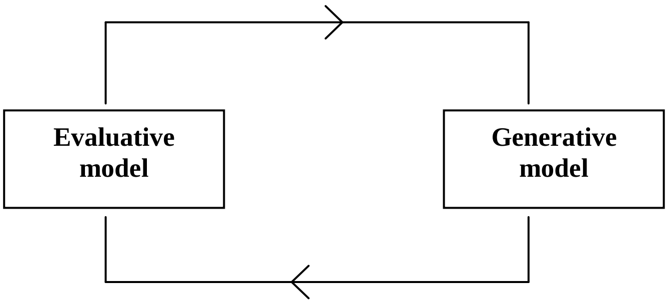
Supply network designing is a strategic issue that affects the whole supply chain performance. There is a close connection between the design and the profitability of a supply chain. In reality these networks are quite complex, they are structured with many facilities and intermediary nodes that are costly. Some times a rational elimination or merging of some of those facilities contributes to cost reduction and improves the total performance of the system. The design of a supply network requires decisions regarding parameters such as the number of suppliers, their capacities, the capacity of production - distribution centre and the capacity of the buffers in order to control the efficient flow of raw materials, in process inventory and finished goods (throughput).

In a supply chain push and pull processes are usually performed. In a push-based supply chain, products are pushed through the supply network from the suppliers to the customers, while in a pull-based supply chain the product flow is based on the consumption of goods in the downstream stages. The interface between the push-based processes and the pull-based processes is known as the push–pull boundary.

The performance of a supply system is affected by the randomness of the stochastic processes that take place such as the supplier’s service rates and the production centre service rate, and by system characteristics such as the number of suppliers and their capacities, the production centre, and the buffer capacity.

The random nature of the processes involved in the behaviour of the system under consideration renders it difficult to obtain important performance measures without a stochastic evaluative model. Evaluative or descriptive models assume a given set of input data and decision variables of the system under study and subsequently the performance measure(s) of the system are evaluated. Such models could be used as generative or prescriptive models (see Figure 1).

Figure 1: The synergistic relationship between evaluative and generative models (Papadopoulos et al., 2010)



In this study a two echelon discrete material, merge supply network is examined. In this network the same part type, after its elaboration, is shipped to a production-distribution centre by two reliable suppliers. The first echelon consists of the distribution centre and a shared buffer, the second echelon includes two non identical reliable suppliers. The paper is organized as follows; section two presents a literature review of the subject. In sections three and four system assumptions and model formulation are laid out. Section five presents illustrative numerical results. Finally section six summarises conclusions and further research.

2 LITERATURE REVIEW

A first variable we can use to classify supply networks is their structure. Based on this there are three generic types of supply networks.

- Linear, if each node-stage receives goods from a single upstream node (supplier) and ships goods to a single downstream node (internal customer).
- Distributive or, divergent or arborescent, if each node receives goods from a single source at the most, but can ship goods to more downstream nodes.
- Assembly or convergent, if each stage-node delivers to a single source but it can receive goods from more than one sources.
- To our best knowledge, a few analytical methods are available in the current literature. Our review starts with references that pertain to discrete material flow systems.
- Altiok and Perros (1986) develop an approximation procedure to decompose split and merge configurations of open networks of queues with blocking.

• Hyo-Seong and Pollock (1989) analyze a merge configuration of open queuing networks with exponential service times and finite buffers. They provide an iterative algorithm to decompose the queuing network into individual queues and analyses each individual queue in isolation.

• Gopalan and Kumar (1994) analyzed a merge production system which has two parallel stations in the first stage followed by a single station in the second stage. The transient behaviour of the system is analyzed and various measures of system performance are evaluated.

• Helber (1998, 1999), deals with merge operations on production systems with a limited buffer capacity and random processing times using the decomposition approach.

• Papadopoulos and Vidalis (2004) analyze a discrete material flow system consisting of three unreliable machines and one buffer of limited capacity. Diamantidis and Papadopoulos (2006) allow the machines to fail not only when they are operational but also when are either blocked or starved.

• Macgregor and Cruz (2005) use an approximation formula for the Buffer Allocation Problem (BAP) in series, merge, and splitting topologies of finite buffer queuing networks.

• Li and Huang (2005) examine a two-product split-merge system consisting of one common main line and two dedicated lines for two part types. The authors develop a method (overlapping decomposition) for evaluating the throughput as a function of the system parameters.

• Bulgak (2006) presents a new approach in optimal interstage buffer allocation problem (BAP) of split-and-merge unpaced open assembly systems. A simulation model developed is used in conjunction with genetic algorithms (GA) to find optimal interstage buffer configurations yielding a maximum production rate. Alternatively an artificial neural network (ANN)-GA approach is used for the same optimization problem (BAP).

• Liu and Li (2009) investigate discrete time split and merge systems with unreliable machines. Buffers of finite capacity are interposed among the machines. They adopt three scheduling policies: circulate, priority and percentage, and present analytical methods to approximate the system production rates of split and merge systems.

• Liu and Li (2010) provide analytical methods to obtain performance analysis of split and merge production systems with exponential machine reliability models, operating under circulate, strictly circulate, priority, and percentage split/merge policies.

For continuous material flow systems we refer to the works of Tan (2001), Helber and Mehtrens (2003), Helber and Hanifa (2004) and Tan and Gershwin (2009). Tan examines a system consisting of two upstream unreliable machines that serve a shared buffer in front of the third machine. He assumes that the combined speed of the machines upstream the buffer is lower than the speed of the downstream machine. Helber and Mehtrens (2003) develop an exact algorithm to compute the throughput for a similar but more general system, which allows for arbitrary deterministic processing times and exponentially distributed failure and repair times. To cope with this, they introduce a priority unblocking rule. Helber and Hanifa (2004) extend the previous approach to more complicated flow lines with unreliable machines and limited buffer capacity. Tan and Gershwin (2009), consider a two stage continuous material flow system separated by a finite capacity buffer. The system is modelled as a continuous time, continuous-discrete state space stochastic process and the steady-state distribution is determined.

In the reported above studies the service or breakdown times are supposed to be exponentials.

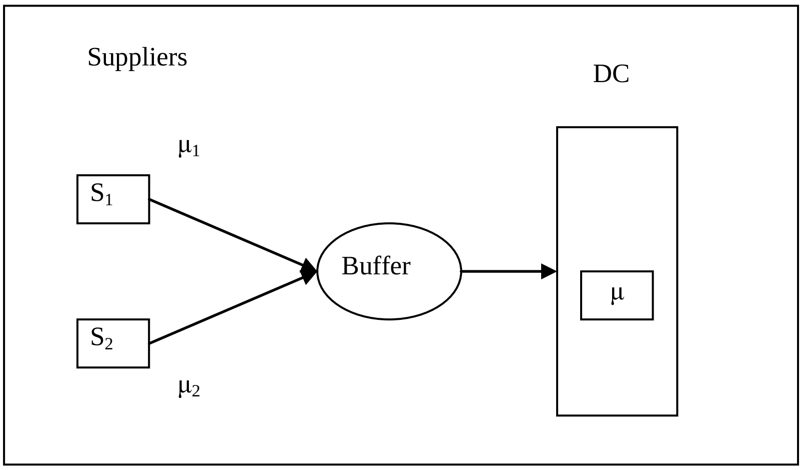
Our research is closer to Diamantidis and Papadopoulos (2004). The system under investigation has the same structure but processing times here are non identical and assumed to be stochastic with Erlang-K for all machines. The contribution of the present work to the research of merge systems is the adoption of Erlang processing times in both echelons.

3 DESCRIPTION OF THE SYSTEM

In this study, we examine a two echelon discrete material flow merge supply network that consists by two suppliers, one buffer and one production-distribution (DC) (see Figure 2). The same type item is shipped by the suppliers to the DC. The actual processing + lead time that the materials are processed by the two suppliers are non identical and follow the Erlang distribution with p_s phases ($p_s \geq 2$) with average rate μ_1 and μ_2 respectively. The production-distribution centre performs another operation on the materials collected in the buffer and the finished products flow outside the system. The processing time in DC is assumed also to follow the Erlang distribution with P_{DC} phases ($P_{DC} \geq 2$). The first echelon consists of the distribution centre and the shared buffer, the second echelon includes two suppliers. This structure can represent a wide range of real systems.

Additionally, there is an unlimited supply of materials to suppliers and an unlimited capacity shipping area after the distribution centre. In other words, suppliers are never starved, and the production-distribution centre is never blocked. Blocking appears when one or more suppliers have finished their process and try to feed the buffer that is full (blocking after service, BAS). The blocking remains until the machine finishes service at DC and one unit from the buffer enters the machine. In case of more than one blockage, on unblocking the supplier with the smallest index has priority. That is suppliers are ranked based on their unblocking priority.

Figure 2: A two stage merge supply network, with two suppliers, a shared buffer and a DC with one machine.



The system under investigation is fully described by the numbers:

K: Number of merged stations (here K=2).
N: Capacity of distribution centre (i.e. number of machines here N=1).
B: Capacity of shared buffer (number of slots).
P_s: The number of Erlang phases for suppliers (here P_s=2).
P_{DC}: The number of Erlang phases for DC (here P_{DC}=2)..
μ_1: Average processing rate of Supplier $_1$.
μ_2: Average processing rate of Supplier $_2$.
μ: Average processing rate of DC.

The main objective of this work is to investigate the influence of each parameter (i.e. P_s, P_{DC}, B, μ_1, μ_2 and μ) on system performance measures. The main contribution of this work is the presentation of an exact evaluative model to calculate the performance measures of a merge push system with Erlang replenishment and processing times. This model can be used as a generative one to determine the values of the parameters that optimize the behaviour of the system given an objective function.

4 METHODOLOGY

Many business problems or procedures can be described by Markov Chain, which stand for modelling uncertainty of adequate real-world dynamic systems. Due to Markovian property the future behaviour of the process depends only on the present state of the process and it is not influenced by its past history.

The Markov process $\{X(t), t \geq 0\}$ is completely determined by the probability distribution of the initial state X_0 and the one-step transition probabilities $p_{ij}(dt)$. In applications of Markov chains the art is (Tijms, 2003):

(a) to choose the state variable(s) such as the Markovian property holds,

(b) to determine the one-step transition probabilities p_{ij}.

In simple words, to model a system as a Markov process the following steps must be implemented: first identify the state space and the possible number of the states, second, create the transition matrix by determining the one-step transition probabilities p_{ij}. Third, derive the steady state probability vector and finally by the steady state probabilities calculate the performance measures of the system.

The merge supply network is analyzed as a continuous time Markov process with a finite number of states. More specifically the system under consideration may be viewed as a birth-death stochastic process. Births correspond to inputs to the buffer and DC and deaths correspond to outputs from the DC. As consequence, the transition matrix is a tri-diagonal matrix. The transition matrix structure is affected by the system parameters: number of suppliers (K), capacity of distribution centre (N) and capacity of shared buffer (B). This structure can be exploited. This leads to the development of a computational algorithm that generates transition matrices for any value of K, B and N. The stationary distribution is used to compute the system's performance for different system characteristics.

The Erlang distribution is selected as a research tool over exponential distribution because it offers lower processing variability, i.e. the variability of processing times diminishes as the number of phases P_s or P_{DC} increases.

4.1 Solution procedure

The steps of the solution method for solving the system under consideration are similar to those applied in Papadopoulos (1989), Papadopoulos and O'Kelly (1989), Papadopoulos, Heavey and O'Kelly (1989a, 1989b), Heavey, Papadopoulos and Browne (1993), Vidalis (1998), Vidalis and Papadopoulos (1999) and Vidalis and Papadopoulos (2001). These steps are described below:

- **Step 1:** Calculate the dimension of the transition matrix as a function of parameters K, P_s, P_{DC}, B and N (here K=2, B=1 and N=1)
- **Step 2:** Generate the transition matrix
- **Step 3:** Calculate the steady-state probability vector and
- **Step 4:** Compute the selected performance measures.

4.2 Illustrative Example

In order to be more understandable, an illustrative example is given for the simple case of a system with one supplier, buffer capacity equal to one and one machine at the DC. The processing times at both echelons are Erlang with 2 phases (P_s=P_{DC}=2). At first we identify the state space and the possible number of the states. The number of states of a system with K suppliers, B slots in buffer and N identical machines in distribution centre is denoted by $S^{K,B,N}$. The number of the states is given by the relationship:

$$S^{K,B,N} = \mathbf{P_s^K + [P_s^K * B + (P_s + 1)^K] * P_{DC}}.$$

The number of states for the system (K=1, B=1, N=1), i.e. the $S^{1,1,1}$ = 2^1+[2^1*1+$(2+1)^1$]*2= 12 possible states for the system. Symbols are used, so as the physical condition of the system is represented. Table 1 shows the interpretation of those symbols. For instance, "b" symbolizes the supplier's blocking when the buffer is full and the DC is busy, "1" or "2" means that the supplier or the DC is in the first or second phase of material processing respectively.

Table 1: System's state representation symbols

Supplier 1or 2	**Buffer**	**DC**
1st erlang phase	0: empty	0: idle
2nd erlang phase	1: full	1st erlang phase
b: blocked	-	2nd erlang phase

Table 2: the possible states of a two stage inventory system with K=1 supplier, B=1 slot, N=1 machine at DC and Erlang -2 processing times at supplier and DC.

DC	B	S_1	states
0	0	1	100
		2	200
		b	-
	1	1	-
		2	-
		b	-
1	0	1	101
		2	201
		b	-
	1	1	111
		2	211
		b	b11
2	0	1	102
		2	202
		b	-
	1	1	112
		2	212
		b	b12

The states of the system are symbolised by a three digit vector (see table 2). The first number indicates the supplier's Erlang phase or blocking (S_1=1, 2 or b), the second number presents the buffer's level (B=0,1) and the third number indicates the machine condition on DC, if it is idle or is in the first or second phase of Erlang processing time (DC=0,1,2). Table 2 presents all the possible combinations between these variables. The 100 state denotes that the supplier is in the 1st phase, the buffer is empty and the DC machine is idle. The b12 state represents the situation that the supplier is blocked, the buffer is full and the DC machine is in the 2nd phase. The supplier can not be blocked while the buffer is empty. So states b00, b01 and b02 are not feasible. Also the DC machine can not be idle while the buffer is full. So the states 110, 210, b10 are also excluded.

Thus, for the merge system K=1, B=1, N=1, the transition matrix has dimension 12x12 (see table 2). Changes in the state of the system at each transition step are caused by the occurrence of one of the following events:

- Completion of the 1st phase at supplier S1 at time Δt with probability $2\mu_1 \cdot \Delta t$
- Completion of the 2nd phase at supplier S1 at time Δt with probability $2\mu_1 \cdot \Delta t$
- Completion of the 1st phase at DC at time Δt with probability $2\mu \cdot \Delta t$
- Completion of the 2nd phase at DC at time Δt with probability $2\mu \cdot \Delta t$

To be more specific, if the system is in state 100, in the next time Δt the supplier's processing phase 1 may be finished with probability rate $2\mu_1 \cdot \Delta t$, so the system jumps to state 200 or remains at the initial state 100 with probability rate $(1\text{-}2\mu_1) \cdot \Delta t$ respectively. The transition matrix is completed following the same concept for the remaining states.

Table 3: The transition matrix of a two stage push merge system K=1, B=1,N=1and Ps=P_{DC}=2.

	100	**200**	**101**	**201**	**111**	**211**	**b11**	**102**	**202**	**112**	**212**	**b12**
100	-2μ1	2μ1										
200		-2μ1	2μ1									
101			-2μ1-2μ	2μ1				2μ				
201				-2μ1-2μ	2μ1				2μ			
111					-2μ1-2μ	2μ1				2μ		
211						-2μ1-2μ	2μ1				2μ	
b11							-2μ					2μ
102	2μ							-2μ1-2μ	2μ1			
202		2μ							-2μ1-2μ	2μ1		
112			2μ							-2μ1-2μ	2μ1	
212				2μ							-2μ1-2μ	2μ1
b12					2μ							-2μ

A computational algorithm in MatLab is developed to generate the transition matrices for different values of buffer capacities. The proposed algorithm then solves the linear system of steady state equations and calculates the steady state probability vector. Once we have calculated the steady-state probabilities, all the performance measures of the system can be estimated. The most important performance measures are the average inventory on system WIP_{system}, the mean output rate or throughput of the system and the mean flow time on system..

The average inventory on system is the mean number of machines that are occupied on DC, the mean number of slots occupied on buffer, and the number of suppliers K.

WIPsystem = Number of suppliers + Mean number of Occupied machines on DC + Mean Buffer Level.

The throughput or mean output rate of the system is given by the relationship:

THR = $\mu \cdot Pr$ [DC is busy]

The WIP_{system} is the mean number of flow units in the system. High levels of WIP_{system} imply increasing costs and vice versa. On the other hand, Throughput is the production performance measure under consideration. Thus, high productivity rates require high Throughput Rates.

5 VALIDATION OF THE MODEL AND NUMERICAL RESULTS

To validate the model, a simulation model in Arena 12.0 has been created to compare the numerical results given by the analytical model. Table 4, shows the results of simulation and the analytical model, which indicates negligible differences between analytical and simulation results. Consequently, the analytical model is verified by simulation, thus the accuracy of numerical results that are given in this section, is ensured.

In this section, some numerical results are presented; thus, the need for validation of the analytical model and the precision of the given results emerges. For that purpose, a simulation model in Arena was created, in order to compare the numerical results.

Table 4: Comparison of results for a system K=2, B=1, N=1, P_s, P_{DC}=2-11 and μ_1=μ_2=μ=1.

	Simulation	Analytical		Simulation	Analytical	
P_s, P_{DC}	THR	THR	Difference	WIP_{system}	WIP_{system}	Difference
2	0,98663	0,98665	0,00002	3,91145	3,91145	0,00000
3	0,99515	0,99515	0,00000	3,94855	3,94855	0,00000
4	0,99809	0,99807	0,00002	3,96762	3,96755	0,00007
5	0,99919	0,99918	0,00001	3,97846	3,97839	0,00007
6	0,99964	0,99964	0,00000	3,98509	3,98502	0,00007
7	0,99983	0,99983	0,00000	3,98938	3,9893	0,00008
8	0,99991	0,99992	0,00001	3,99225	3,99217	0,00008
9	0,99995	0,99996	0,00001	3,99425	3,99417	0,00008
10	0,99997	0,99998	0,00001	3,99566	3,99559	0,00007
11	0,99998	0,99999	0,00001	3,9967	3,99663	0,00007

Figures 3, 4 represent the values for Throughput and WIP_{system} respectively from analytical and simulations models. There is almost perfect matching, that indicates the accuracy of the analytical model.

Figure 3: Comparison of throughput for the system K=2, B=1, N=1, P_s, P_{DC}=2-11 and μ1=μ2=μ=1.

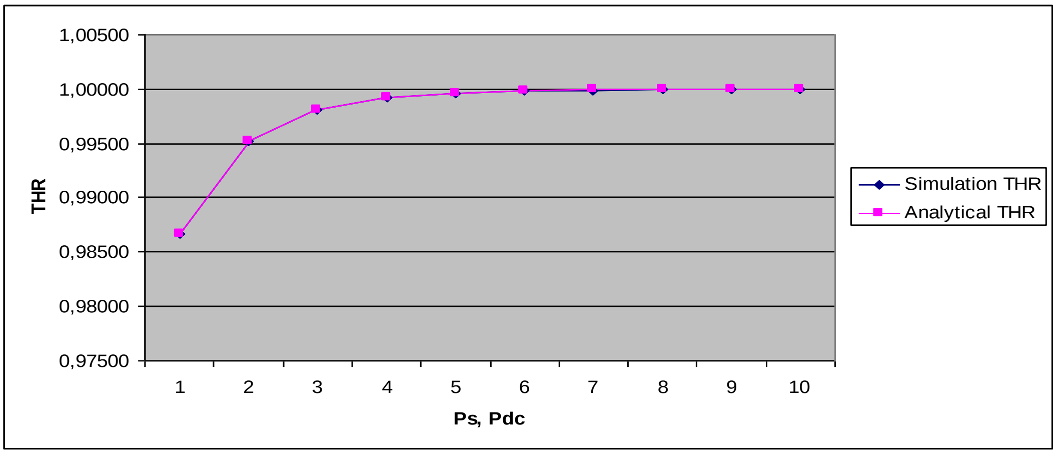
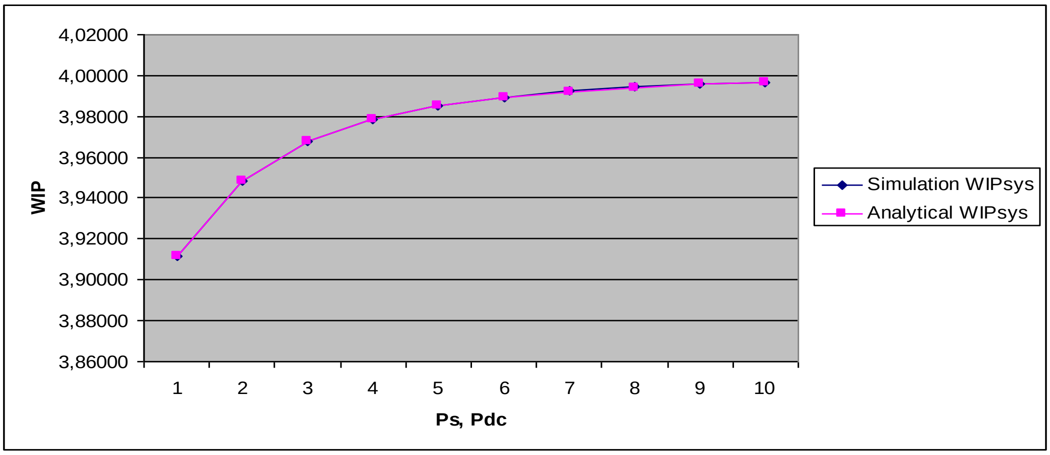


Figure 4: Comparison of WIP_{system} for the system K=2, B=1, N=1, P_s P_{DC}=2-11 and μ_1=μ_2=μ=1.



6 NUMERICAL RESULTS

In this Section, some sample numerical results, are provided. In Sub-section 6.1 the effect of the number of phases P_s and P_{DC} on Throughput and WIP_{system} is examined**,** while in Sub-section 6.2 the effect of supplier's processing rates μ_1 and μ_2 on performance measures is investigated. Finally Sub-section 6.3 analyzes the effect of DC's processing rate on system performance measures.

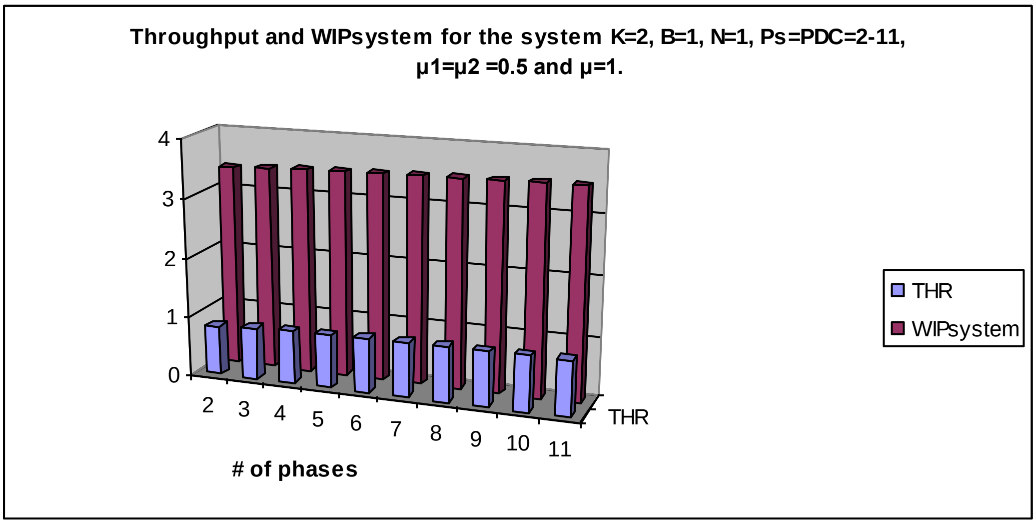
6.1 Effect of the number of phases P_s and P_{DC} on Throughput and WIP_{system}

Firstly, the effect of Supplier's and DC's erlang processing times on performance measures is examined. Table 5 and Figure 5 represent the evolution of Throughput and WIP_{system}, when P_s and P_{dc} scale from 2 to 11, while supplier's rate are μ_1=μ_2= 0.5, and DC rate is μ=1 that is the system is balanced. The inbound rate to the buffer is equal to the outbound rate. As the number of phases (P_s and P_{DC}) increases in both replenishment times and service times so Throughput and WIP_{system} also increase in a diminished rate. The throughput increases tending to its upper bound due to the reduction of processing time variability. Also WIP_{system} increases but the effect of the increment of phases P_s and P_{DC} is smaller comparing with throughput's augmentation.

Table 5: Throughput and WIP_{system} for a system K=2, B=1, N=1, P_s, P_{DC}=2-11 , μ_1,μ_2 =0.5 and μ=1.

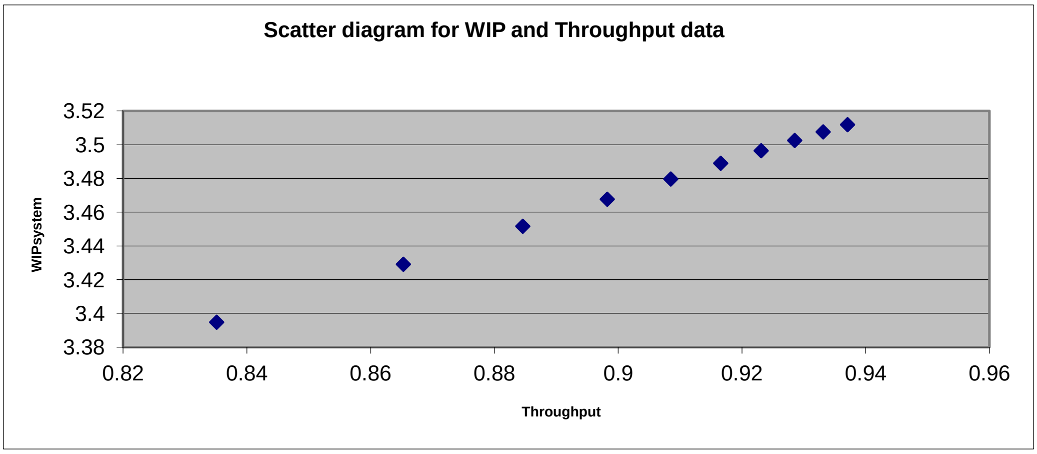
P_s, P_{DC}	THR	WIP_{system}
2	0,83511	3,39475
3	0,86529	3,42911
4	0,88459	3,45164
5	0,89824	3,46766
6	0,90851	3,47966
7	0,91658	3,48899
8	0,92312	3,49644
9	0,92854	3,50252
10	0,93314	3,50759
11	0,93708	3,51187

Figure 5: Throughput and WIP_{system} for the system K=2, B=1, N=1, P_s=P_{DC}=2-11, μ_1=μ_2 =0.5 and μ=1.



A question is placed: is there any relationship between throughput and WIP_{system}'s increment in a balanced merge system? The scatter diagram in Figure 6 below indicates that there is a connection between the two variables.

Figure 6: Scatter diagram for WIPsystem and Throughput data



Regression Analysis describes as closely as possible the relationship between throughput and WIP_{system}. So we can predict what value the dependent variable (WIP_{system}) will assume, given specific value for the independent variable (throughput). For the values from Table 5 regression analysis gives the relationship

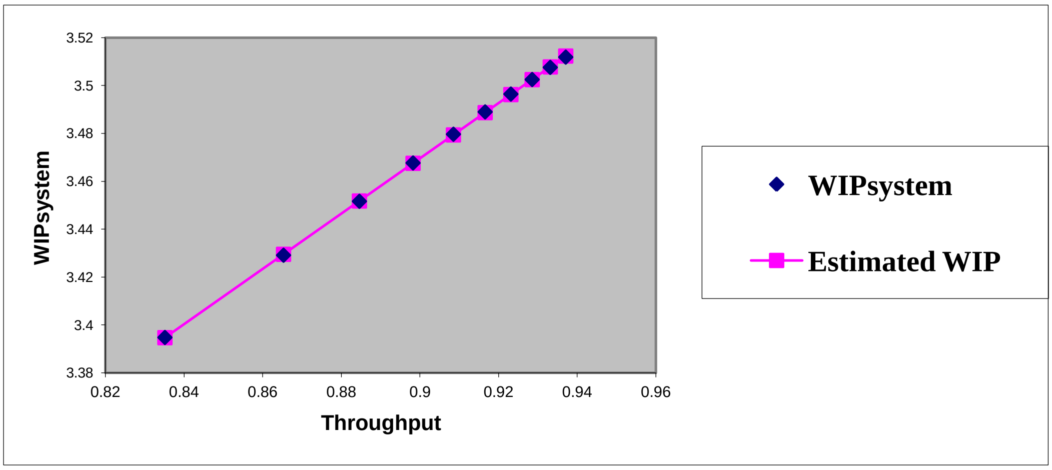
$$\boxed{WIP_{system}=\mathbf{1{,}153921} \cdot \text{Throughput} + \mathbf{2{,}431009}} \quad (*)$$

e.g. for throughput =0,92312 we have by (*)

$$WIP_{system}=1{,}153921 * 0{,}92312 + 2{,}431009 = 3{,}496217$$

The R^2 statistic is 0,99997. Because this value is close to the maximum possible R^2 value (1) this statistic indicates that the regression function we have estimated fits our data well. Also the standard error S_e is equal to 0,00029 indicating a good accuracy of the prediction obtained from the regression model.

Figure 7: WIP_{system} evolution as a function of Throughput



6.2 Effect of supplier's processing rate μ_1 and μ_2 on performance measures

Additionally, the effect of supplier's processing rate μ_1 and μ_2 on performance measures is investigated (see table 6, Figure 8). The processing rates of suppliers, μ_1 and μ_2 increase from 0.2-1.8, while the processing rate at DC remains stable, μ=1, and the number of phases P_s and P_{DC} are constant (equal to 4). As the processing rates μ_1, μ_2 are growing, throughput is growing too. When μ_1 end μ_2 reach the value 0.8, Throughput catches a slow growing rate. The explanation is that in this value the inbound rate is greater than the outbound rate so the system's output tends to its upper value that is that is the processing rate of the bottleneck station (here the DC) WIP_{system} follows a high growing rate for all values of μ_1, μ_2 from 0,2-0,8.

Table 6: Throughput and $WIPs_{ystem}$ for a system K=2, B=1, N=1, P_s=P_{DC}=4, μ_1, μ_2=0.2-1.8 and μ=1.

μ_1,μ_2	THR	WIP_{system}
0,2	0,39962	2,45932
0,4	0,76495	3,11731
0,6	0,94927	3,68973
0,8	0,99073	3,90499
1,0	0,99807	3,96755
1,2	0,99952	3,98738
1,4	0,99986	3,99455
1,6	0,99995	3,99744
1,8	0,99998	3,99872

Figure 8: Throughput and WIP_{system} for a system K=2, B=1, N=1, P_s, P_{DC}=4, μ_1, μ_2=0.2-1.8 and μ=1,5

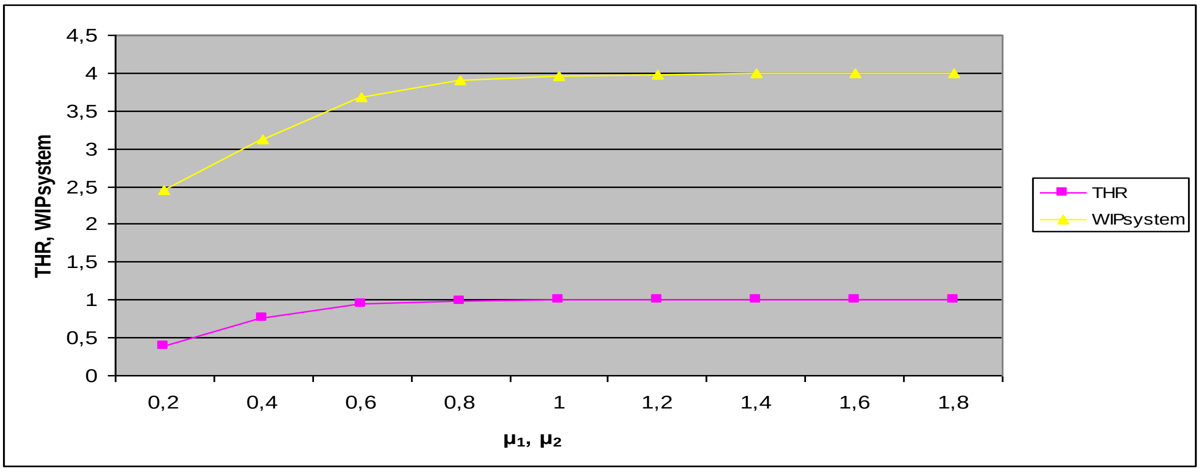


Figure 9: WIP_{system} evolution as a function of throughput

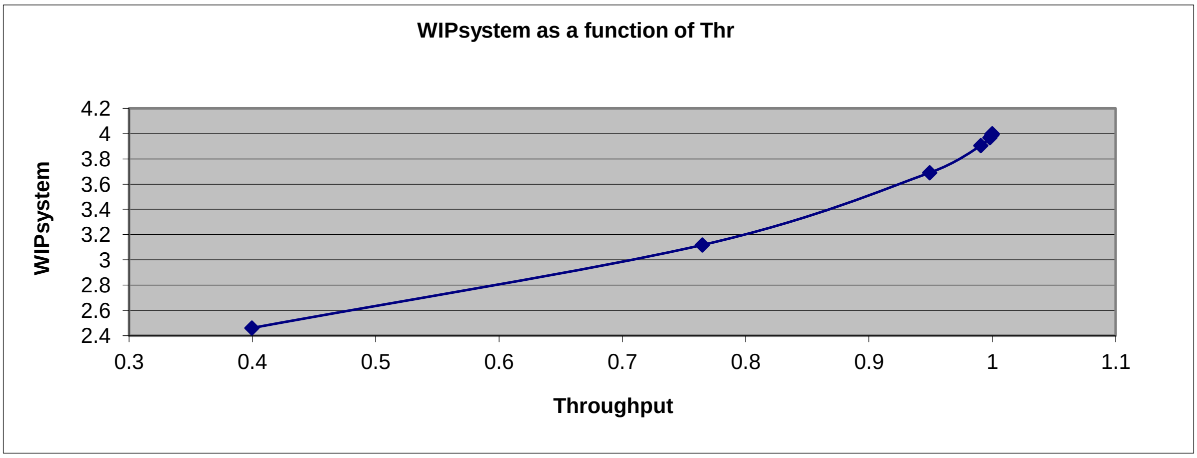


Figure 9 indicates that WIP_{system}'s increment is greater than the corresponding of Throughput.

6.3 Effect of DC's processing rate on performance measures

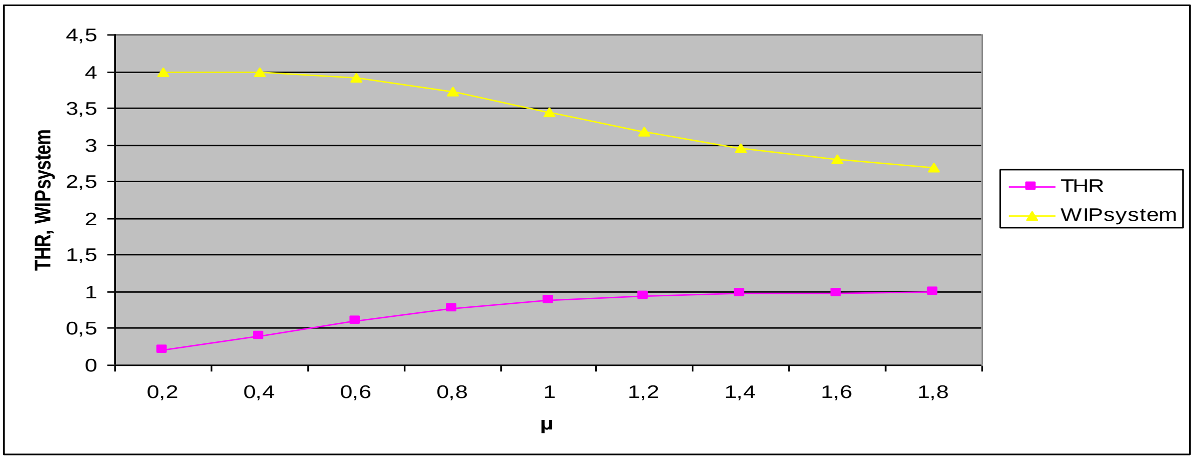
Finally, the impact of DC's processing rate on performance measures is examined. The value of μ scales from 0.2 to 1.8, while μ_1, μ_2, P_s and P_{DC} are constant and equal to 0.5 (table 7, Figure 10). In that case, as μ value increases, Throughput increases therefore, the productivity of the systems increases too. On the other hand, as μ value increases, WIP_{system} declines so, the holding cost declines as well. The explanation is: for the values of μ, $0.2 \leq \mu \leq 0.8$ the bottleneck station is the DC (the inbound supply rate is greater than the outbound rate of the DC). At the point of μ=1 the system becomes balanced. For the rest values of μ, $1.2 \leq \mu \leq 1.8$ the DC outbound rate is greater than the supply inbound rate. As DC rate increases also throughput increases until its upper limit

(=1) because blocking is decreased. WIP_{system} decreases because as DC becomes faster blocking and buffer utilization are decreased.

Table 7: Throughput and WIP_{system} for the system K=2, B=1, N=1, $P_s=P_{DC}=4$, $\mu_1=\mu_2=0,5$ and $\mu=0,2\text{-}1,8$.

μ	THR	WIP_{system}
0,20	0,1999	3,9998
0,40	0,3998	3,9898
0,60	0,5957	3,9213
0,80	0,7672	3,7327
1,00	0,8845	3,4516
1,20	0,9467	3,1772
1,40	0,9752	2,9618
1,60	0,9879	2,8042
1,80	0,9937	2,6889

Figure 10: Throughput and WIP_{system} for a system K=2, B=1, N=1, P_s, $P_{DC}=4$, μ_1, $\mu_2=0,5$ and $\mu=0,2\text{-}1,8$.



7 CONCLUSIONS AND FURTHER RESEARCH

In this work, two non identical reliable suppliers feed a DC with a shared buffer. The DC has a reliable machine that proceeds the material. The system was analyzed as a continuous time Markov process with discrete states. The structure of the transition matrix was investigated and a computational algorithm is developed to generate stationary distribution for two suppliers (K=2), buffer capacity equal to one (B=1) and one machine in DC (N=1). The input rates (active processing time + replenishment times) are non identically random variables erlangian distributed. The algorithm is used to explore the influence of each or combination of parameters on system performance measures (Throughput and WIP_{system}).

The main findings of this work may be summarized as follows:

- As the number of phases (P_s and P_{DC}) increases in both replenishment times and service times so throughput and WIP_{system} also increase in a diminished rate in a balanced merge system.
- As the number of phases (P_s and P_{DC}) increases in both replenishment times and service times throughput's increment is more intense.
- Regression Analysis describes a strictly linear relationship between throughput and WIP_{system} in a balanced merge system.
- As the processing rates μ_1, μ_2 are growing, throughput is growing too. When μ_1 end μ_2 reach the value 0,8, Throughput catches a slow growing rate. WIP_{system} follows a high growing rate for all values of μ_1, μ_2 from 0,2-0,8
- As DC rate μ increases, Throughput increases therefore, the productivity of the systems increases too. On the other hand, as μ value increases, WIP_{system} declines.

As possible areas for further research are suggested:

- The system may be extended according to the number of suppliers, the buffer capacity and number of machines in DC.
- Suppliers may have different capacities (more than one machine)

- Active processing time + replenishment time may follow another phase type distribution
- Suppliers may be unreliable
- Machines at DC may be unreliable

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